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Application of Photon Statistics to the Quanta Image Sensor

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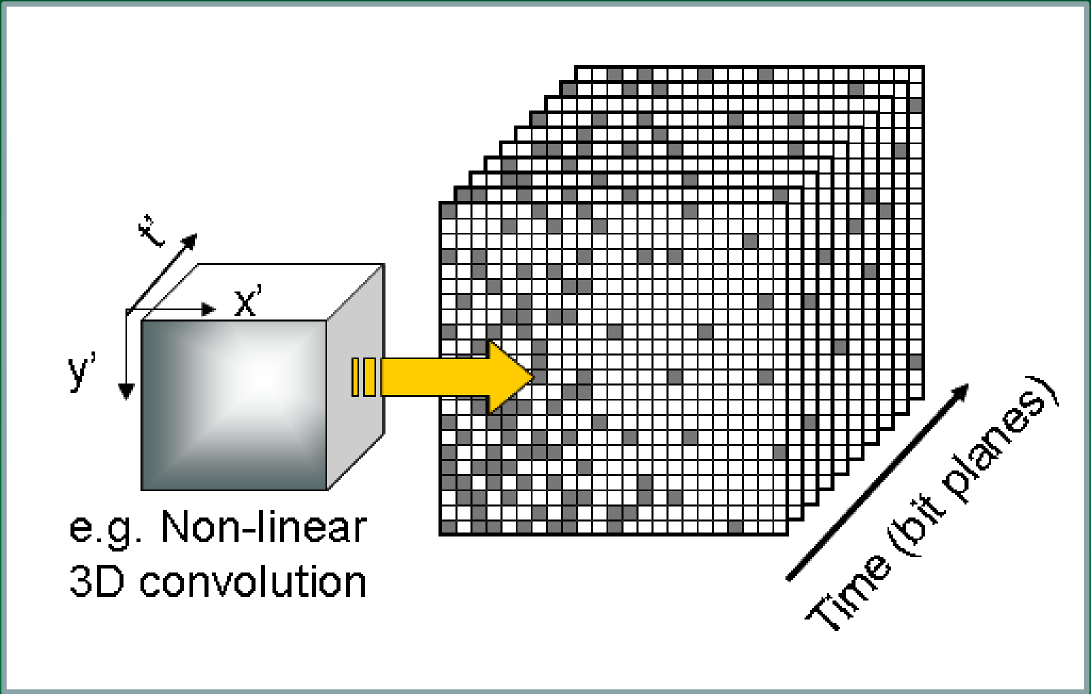
June 15, 2013

Quanta Image Sensor

Jot = specialized SDL pixel, sensitive to a single photoelectron with binary output, “0” for no photoelectron, “1” for at least one photoelectron.



Many jots are needed to create a single image pixel.

e.g. 16x16x16 = 4,096

A QIS might have 1G jots, read out at 1000 fields/sec or 0.5 Tbits/sec

Photon and photoelectron arrival rate described by Poisson process

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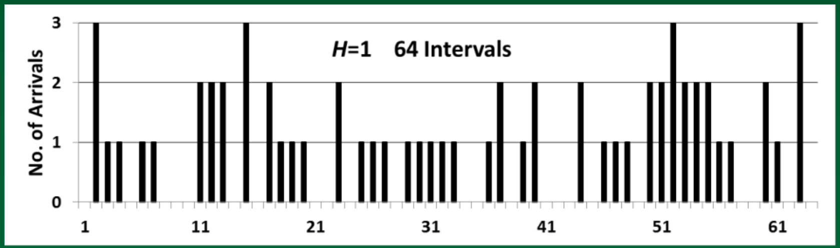
Define *quanta exposure* $H = \phi\,\tau$ H = 1 means expect 1 arrival on average.

Probability of k arrivals

$$P[k] = \frac{e^{-H}\,H^k}{k!}$$

Monte Carlo



For jot, only two states of interest

$$P[0] = e^{-H}$$

$$P[k>0] = 1 - P[0] = 1 - e^{-H}$$

For ensemble of *M* jots, the expected number of 1's : $M_1 = M \cdot P[k>0]$

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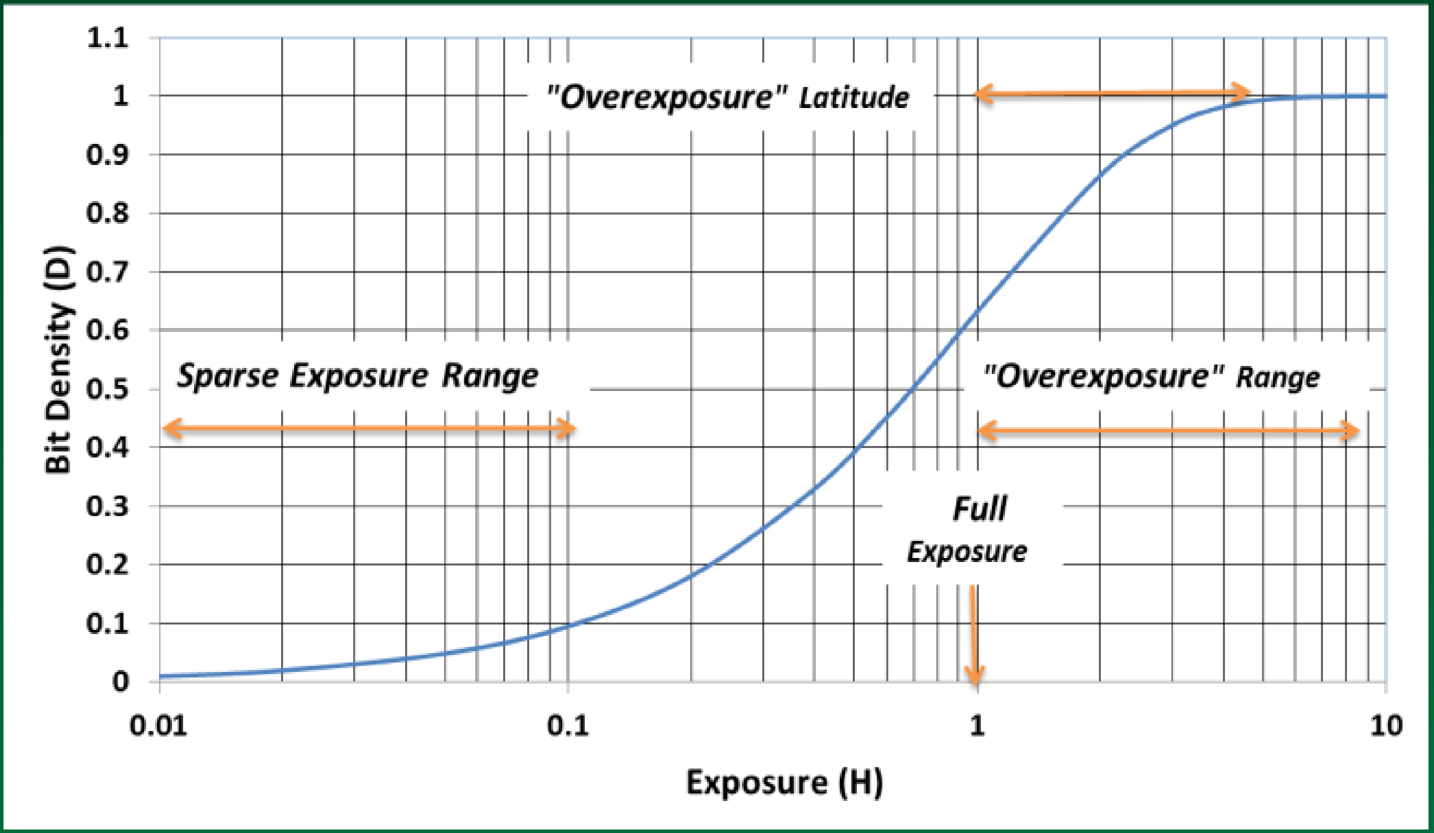
Bit Density

$$Bit\ Density\ D \triangleq \frac{M_1}{M} = 1 - e^{-H}$$



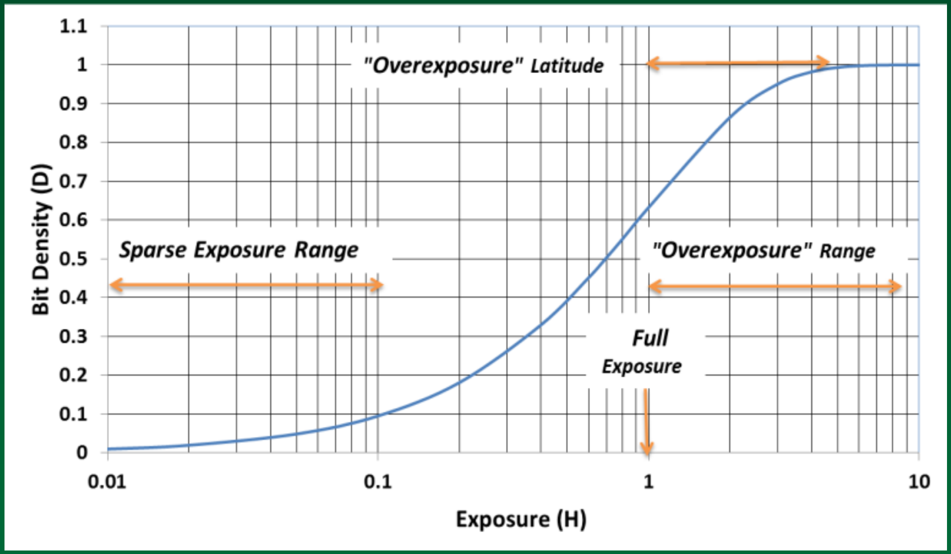
$$\underbrace{\qquad\qquad\qquad}_{D \cong H \quad \text{(linear)}}$$

Can determine H from measured D

$$\mathrm{H} = \ln\left[\frac{1}{1-D}\right]$$

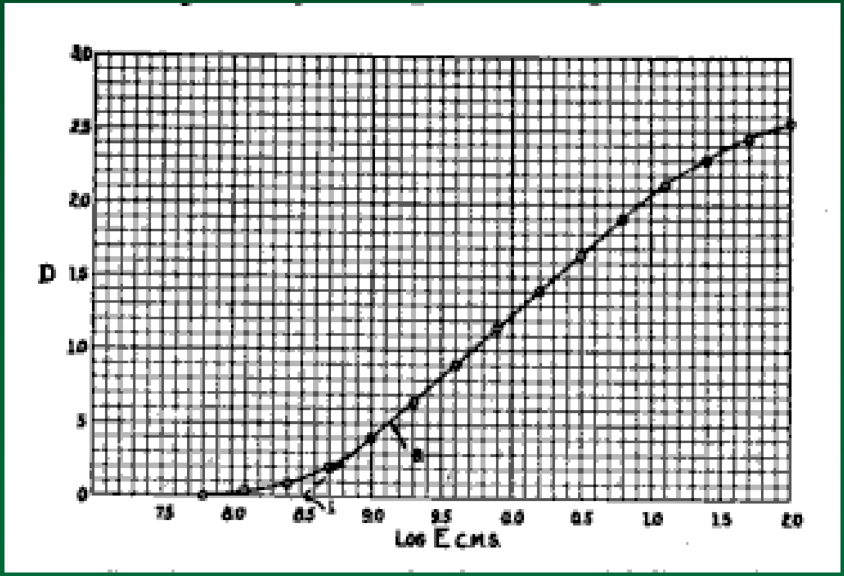
Film-like Exposure Characteristic

QIS D – log H



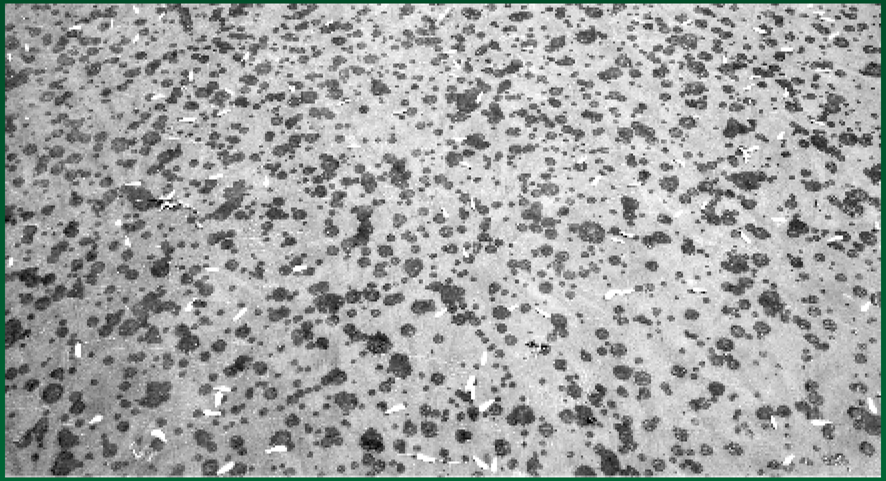
Bit Density vs. Exposure

Film D – log H



Film Density vs. Exposure
1890 Hurter and Driffield

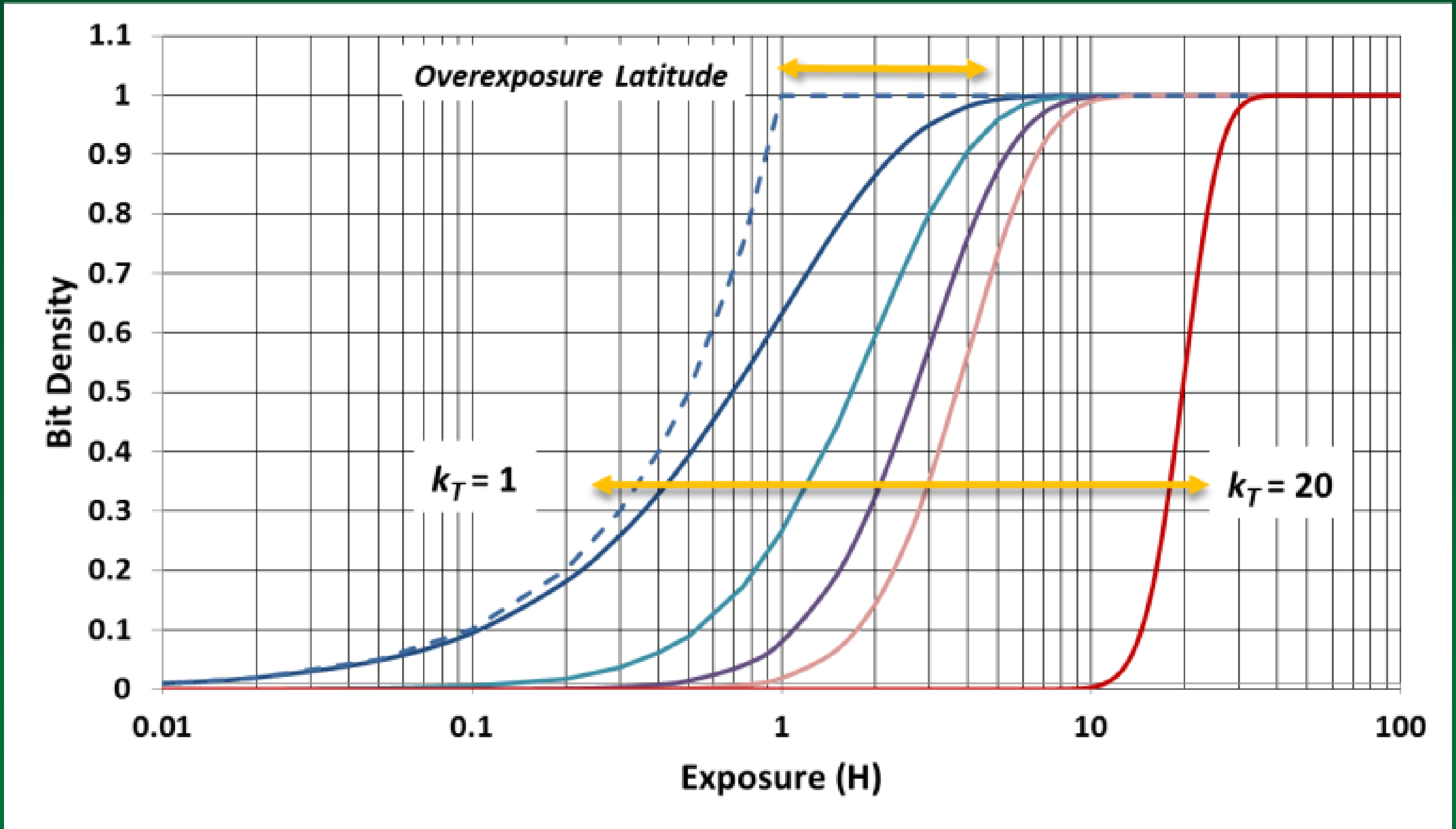
Raindrops on Ground



H~ 0.3 ?

Multi-Arrival Threshold (Not QIS)

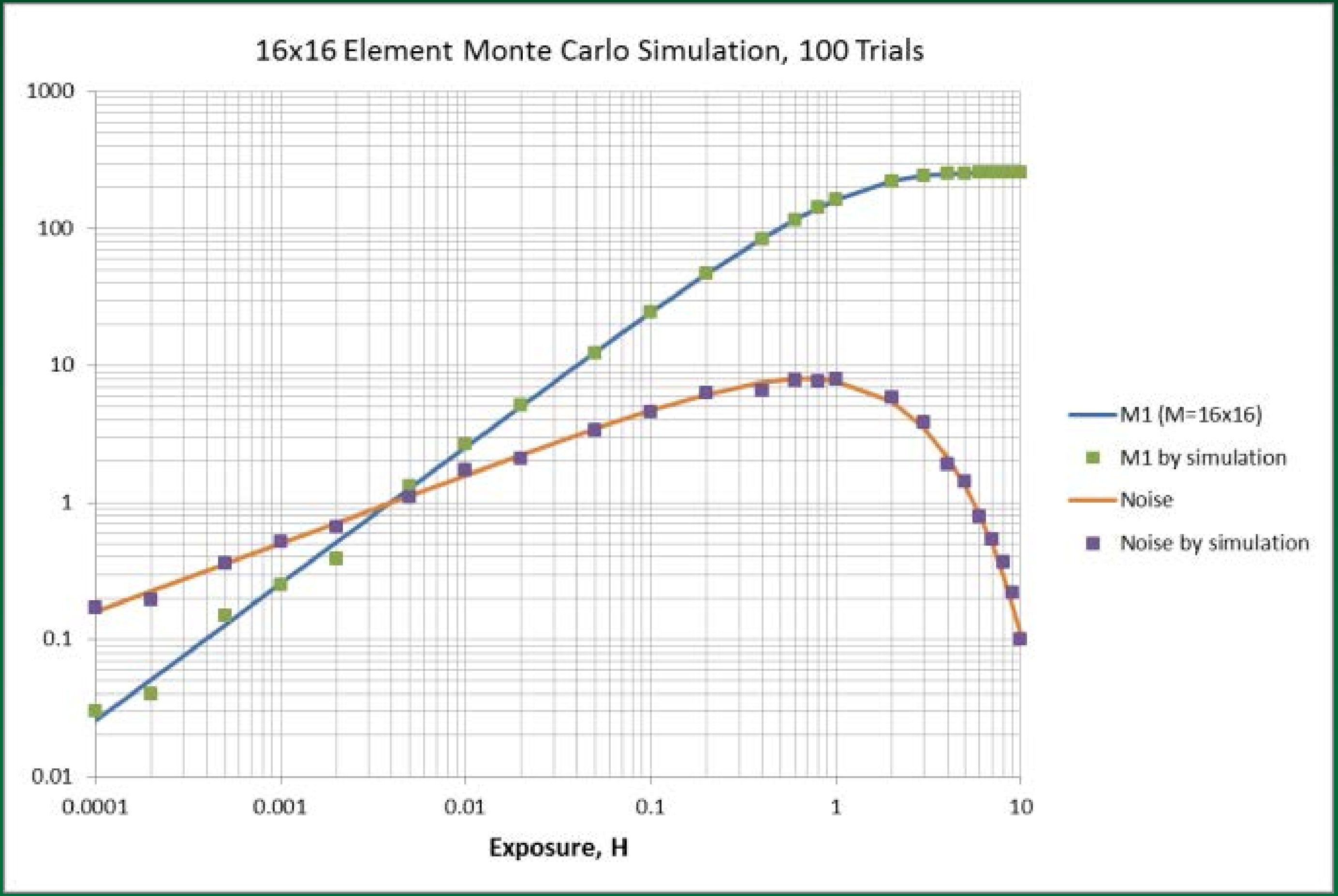
Binary output of sensor ="1" when # of arrivals $k \geq k_T$
Results in reduced higher slope and less overexposure latitude



“Shot” Noise

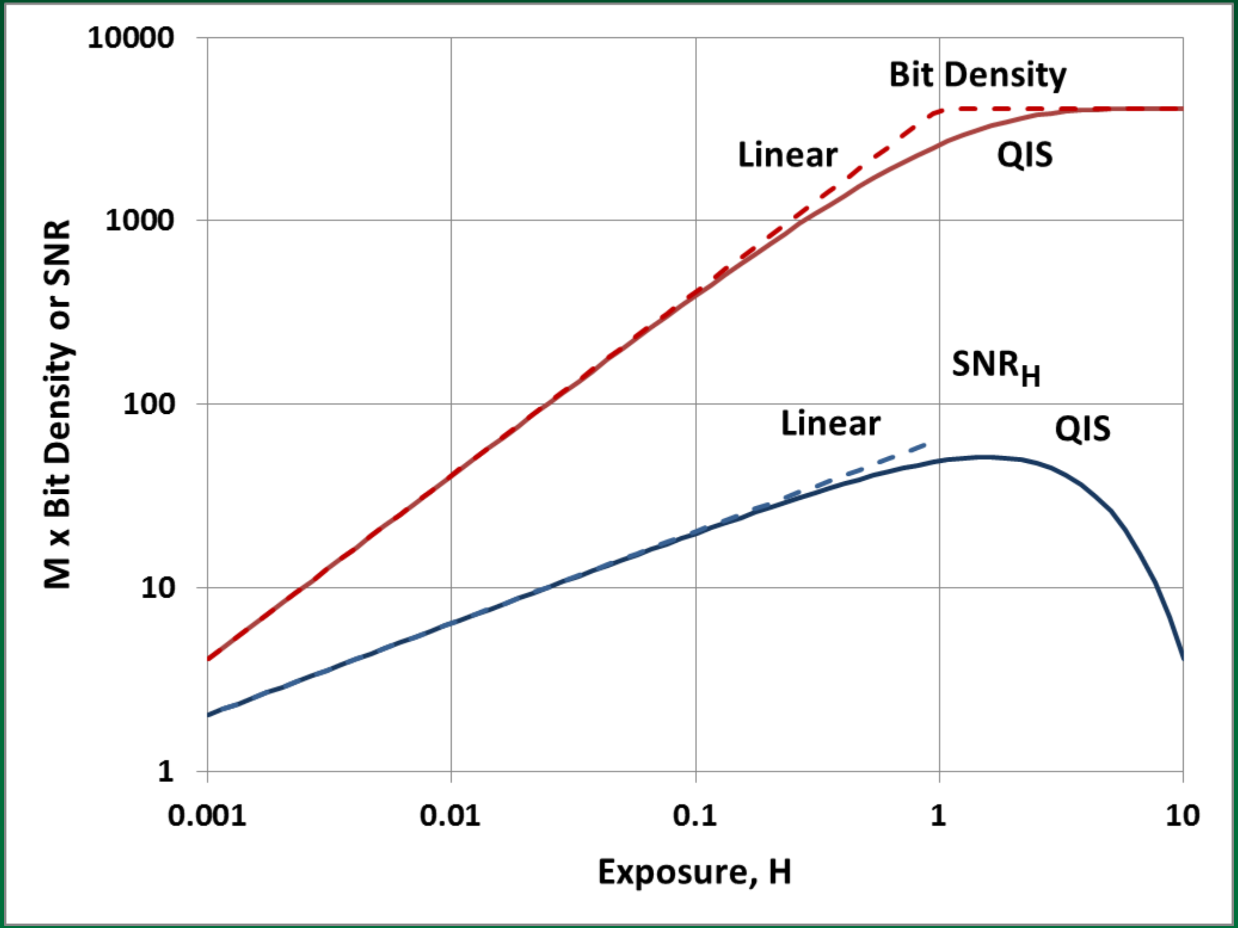
Variance of a binomial distribution

$$\sigma_1{}^2 = M \cdot P[0] \cdot P[k > 0]$$



Exposure-Referred Noise

$$\sigma_H = \sigma_1 \frac{dH}{dM_1} \qquad SNR_H = \frac{H}{\sigma_H} = \sqrt{M}\frac{H}{\sqrt{e^H - 1}}$$



M=4096

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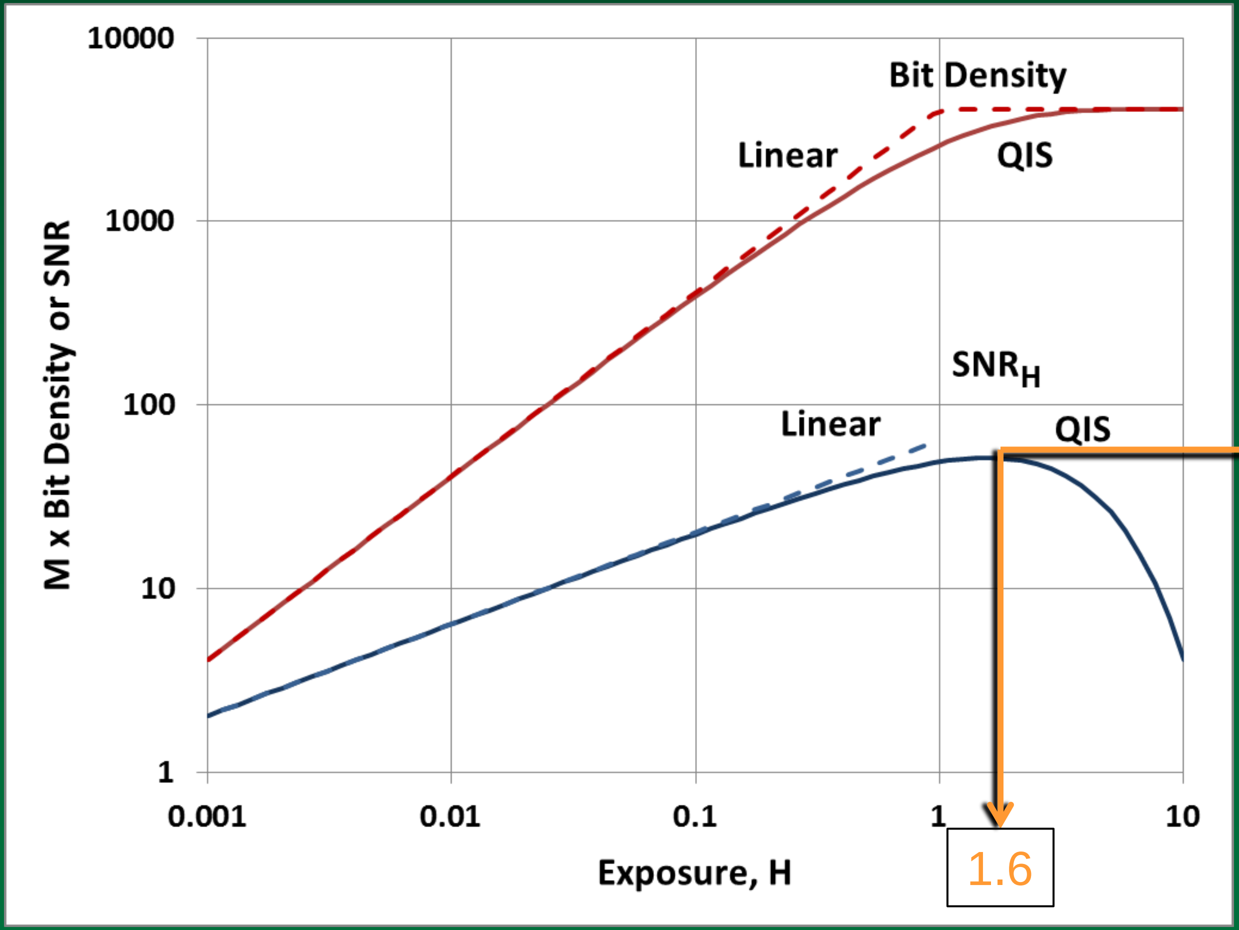

M=4096

51.5
34.2 dB

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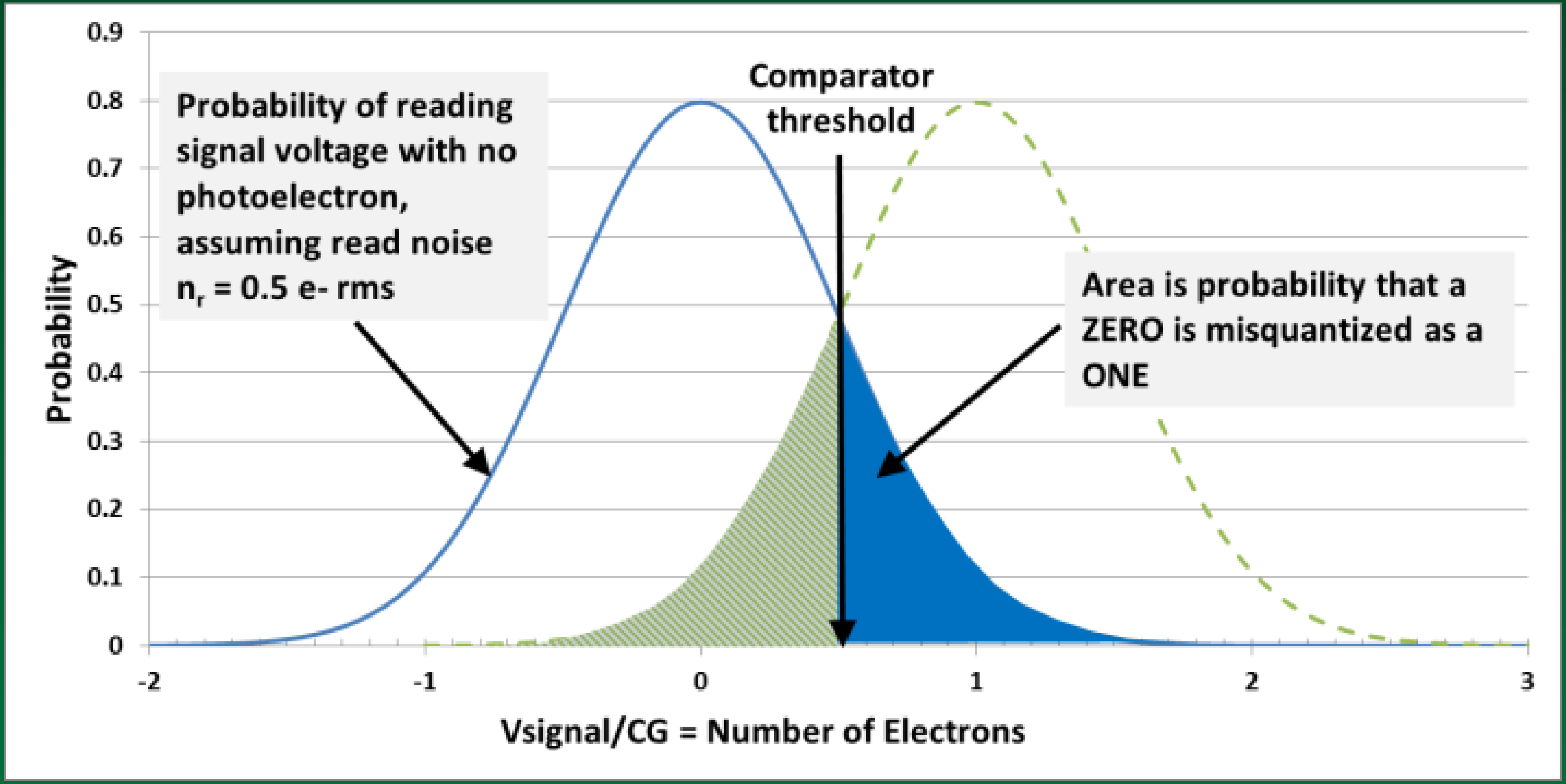
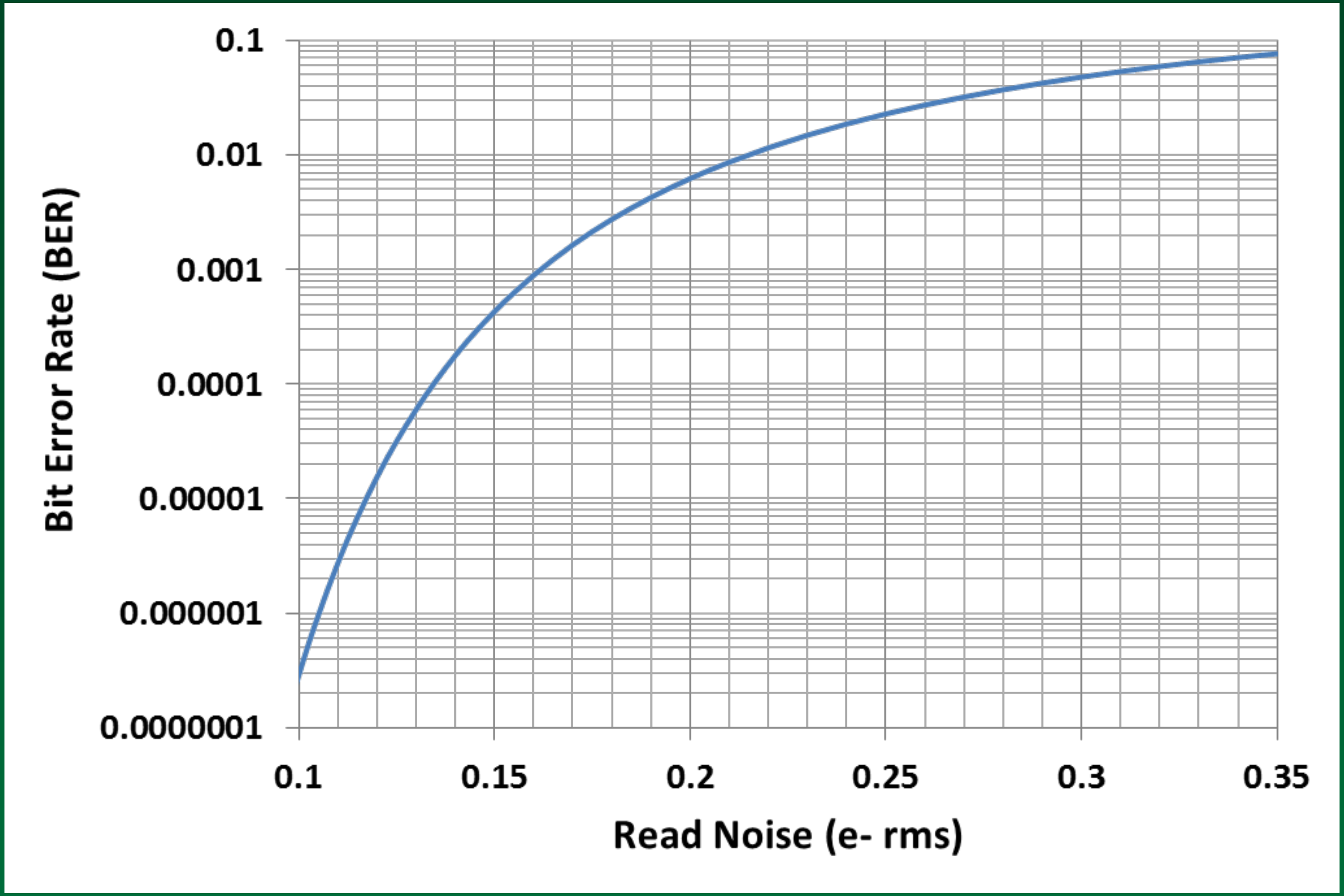
Read Noise and Bit Error Rate (BER)



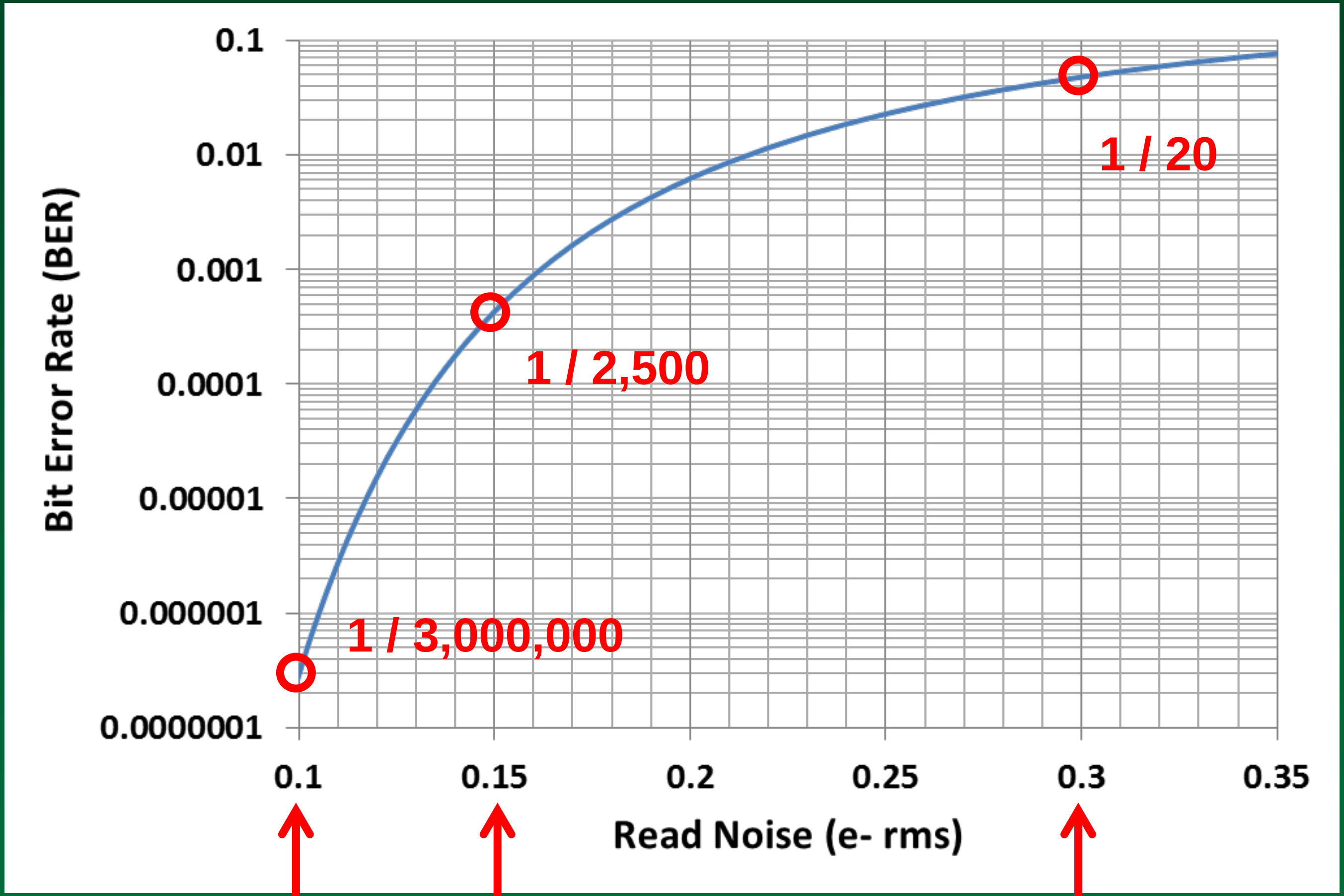
BER vs. Read Noise



$$BER = \frac{1}{2} erfc\left(\frac{1}{\sqrt{8 n_r}}\right)$$

What is an acceptable bit error rate?

BER vs. Read Noise



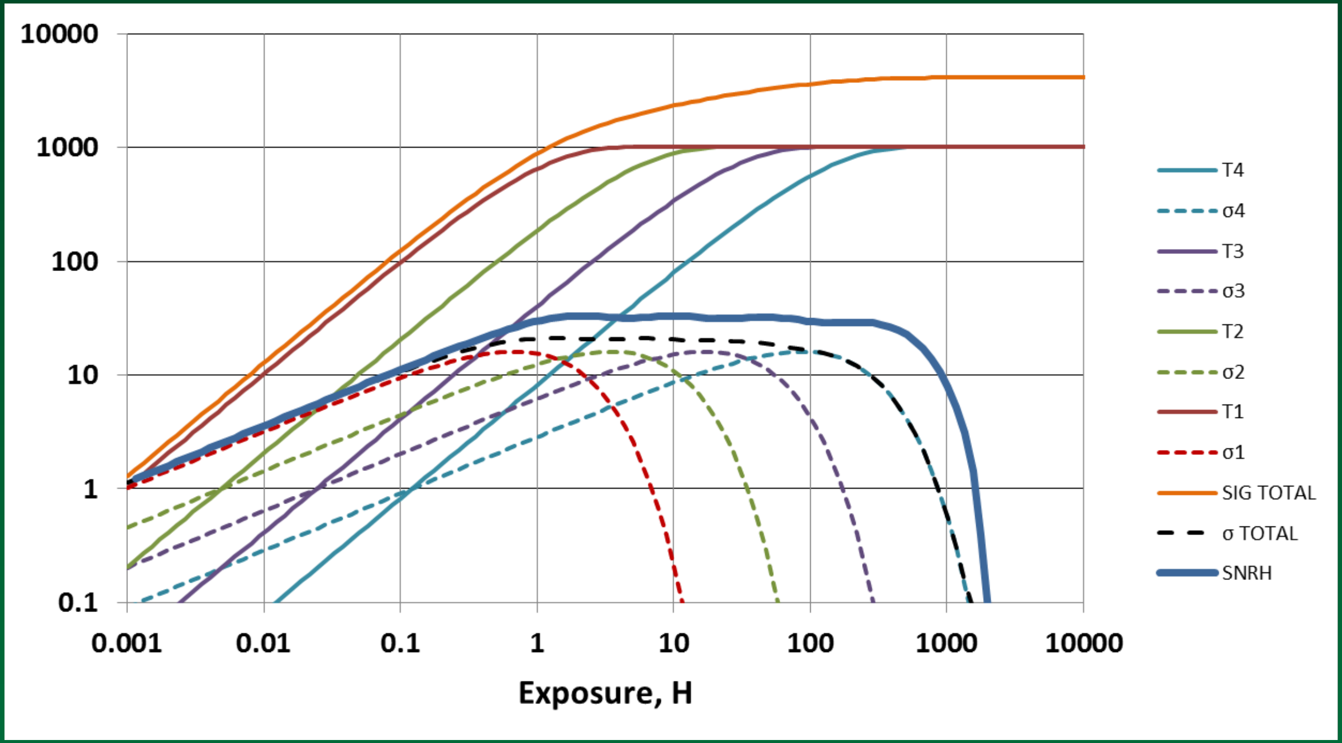
Fossum 2011 WAG

Fossum 2013

Teranishi 2012

Increased Dynamic Range

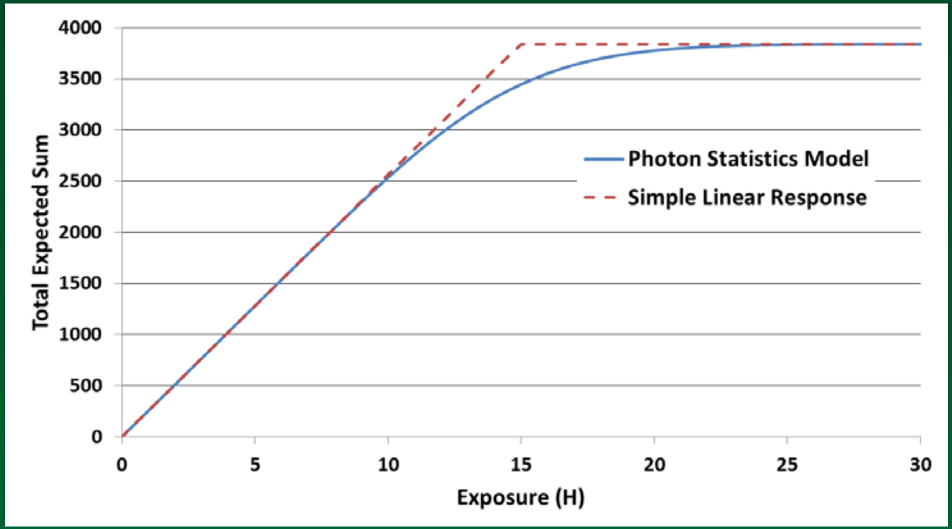
Sum of 16 fields
4@ T=1.0
4@ T=0.2
4@ T=0.04
4@ T=0.008



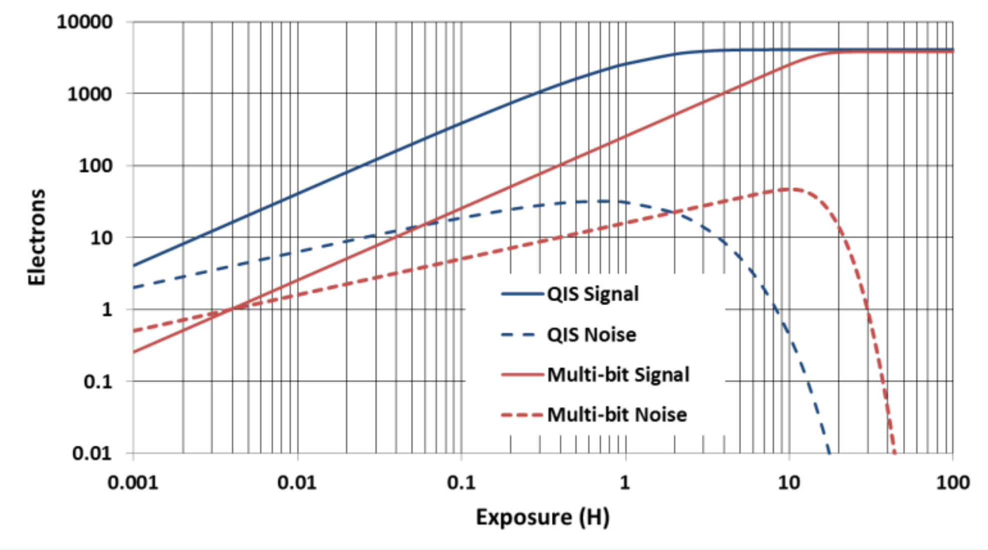
120 dB

Multi-bit Pixels

Counting low number of photoelectrons, e.g. 4b yields FW = 15 e-



Sum 4x4x16 = 256 pixels
Max = 15x256 = 3840



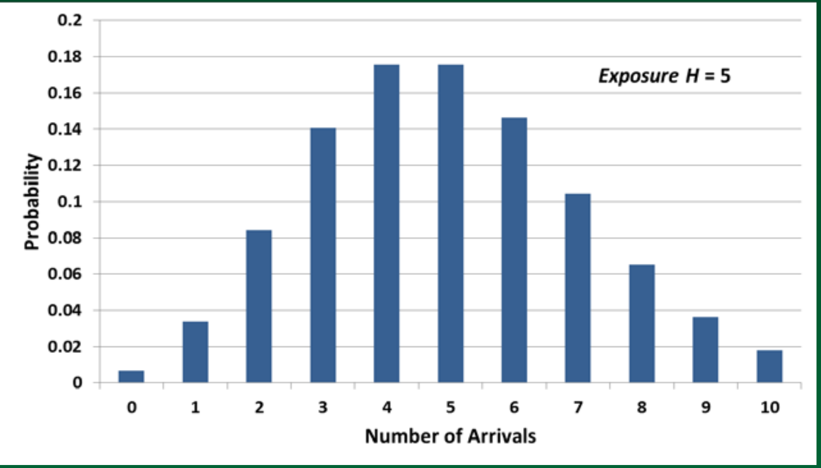
1b v. 4b

QIS: M=4096
4b: M=273

Shot Noise and Read Noise

"Shot" Noise

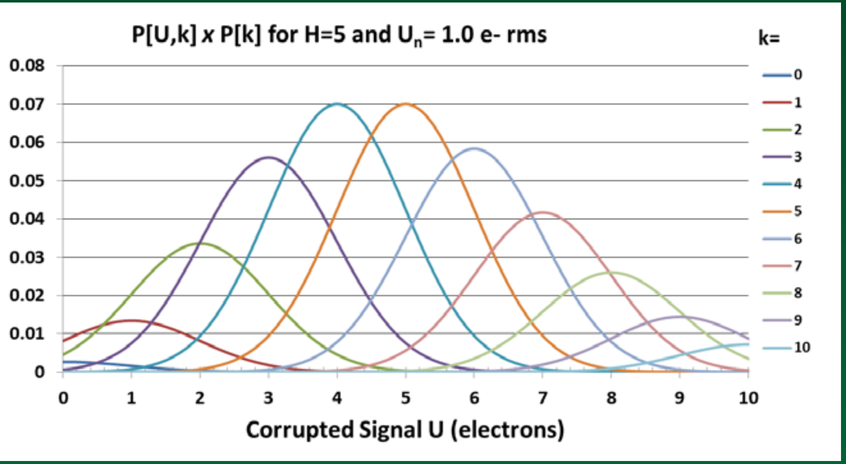
$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$$

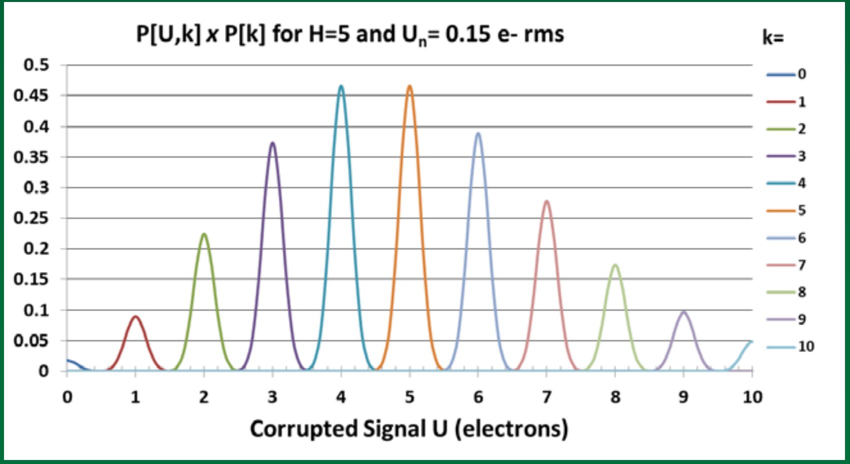


$$P[k] = \frac{e^{-H} H^k}{k!}$$

plus

Read Noise
(Gaussian model)





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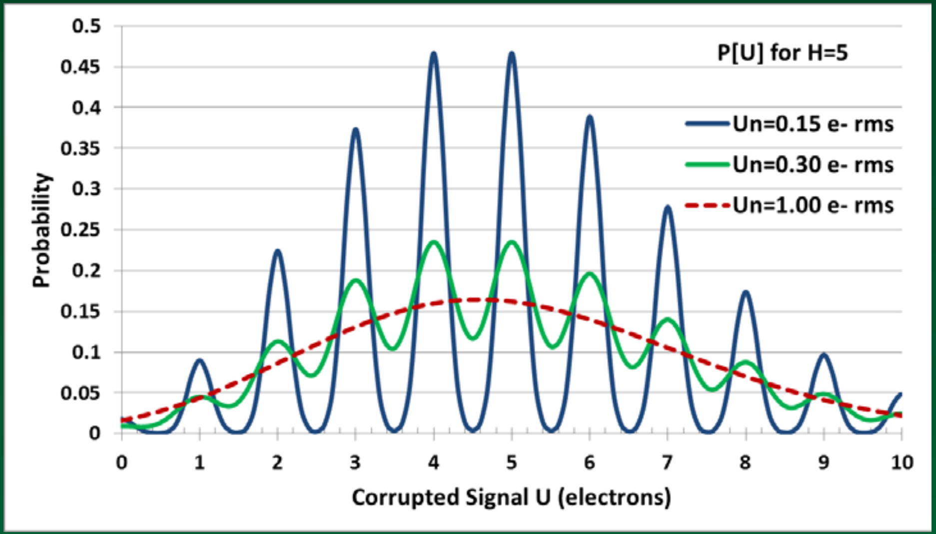
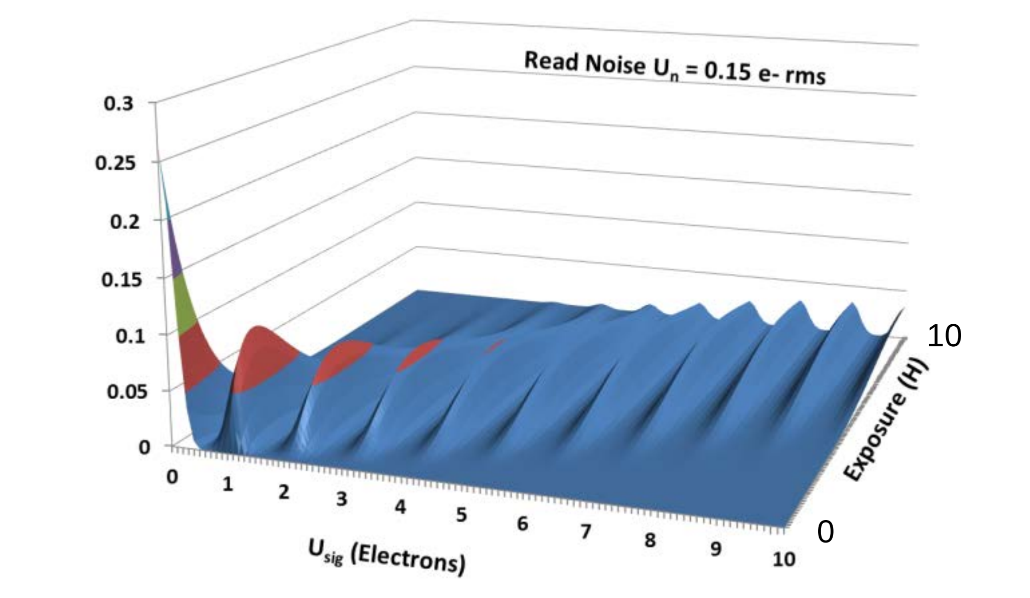
Effect of Read Noise on Photoelectron Counting for Multi-bit Pixel



Note “peak” for H=5 is not at 5 e-



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Summary

- Introduced concept of quanta exposure
- Quantified D-log H response
    - No adjustable parameters – should be very reproducible sensor to sensor.
    - Can retrieve H reliably over large range including non-linear portion ($0.1 < H < 5$)
- Quantified “shot” noise in QIS
- Effect of read noise on BER analyzed
- Extended DR with almost flat SNR discussed
- Multi-arrival and multi-bit pixels also addressed.
- More details in paper submitted to JEDS.