Application of Photon Statistics to the Quanta Image Sensor

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Jot = specialized SDL pixel, sensitive to a single photoelectron with binary output, “0” for no photoelectron, “1” for at least one photoelectron.

Many jots are needed to create a single image pixel.

e.g. $16 \times 16 \times 16 = 4,096$

A QIS might have 1G jots, read out at 1000 fields/sec or 0.5 Tbits/sec
Photon and photoelectron arrival rate described by Poisson process

Define \( quanta \ exposure \ H = \phi \tau \) \( H = 1 \) means expect 1 arrival on average.

Probability of \( k \) arrivals

\[
P[k] = \frac{e^{-H} H^k}{k!}
\]

For jot, only two states of interest

\[
P[0] = e^{-H}
\]

\[
P[k > 0] = 1 - P[0] = 1 - e^{-H}
\]

For ensemble of \( M \) jots, the expected number of 1’s : \( M_1 = M \cdot P[k > 0] \)
Bit Density

Bit Density \( D \triangleq \frac{M_1}{M} = 1 - e^{-H} \)

Can determine \( H \) from measured \( D \)

\[ D \approx H \quad \text{(linear)} \]

\[ H = \ln \left[ \frac{1}{1 - D} \right] \]
Film-like Exposure Characteristic

QIS D – log H

Film D – log H

Bit Density vs. Exposure

Film Density vs. Exposure
1890 Hurter and Driffield
Raindrops on Ground

$H \sim 0.3$
Multi-Arrival Threshold (Not QIS)

Binary output of sensor = “1” when # of arrivals $k \geq k_T$
Results in reduced higher slope and less overexposure latitude
"Shot" Noise

Variance of a binomial distribution

\[ \sigma_1^2 = M \cdot P[0] \cdot P[k > 0] \]
Exposure-Referred Noise

\[
\sigma_H = \sigma_1 \frac{dH}{dM_1}
\]

\[
SNR_H = \frac{H}{\sigma_H} = \sqrt{M} \frac{H}{\sqrt{e^H - 1}}
\]

\[M=4096\]
Exposure-Referred Noise

\[ \sigma_H = \sigma_1 \frac{dH}{dM_1} \]

\[ SNR_H = \frac{H}{\sigma_H} = \sqrt{M} \frac{H}{\sqrt{e^H - 1}} \]

\[ M = 4096 \]

51.5 dB

34.2 dB
Read Noise and Bit Error Rate (BER)

- Probability of reading signal voltage with no photoelectron, assuming read noise $n_r = 0.5$ e- rms
- Comparator threshold
- Area is probability that a ZERO is misquantized as a ONE

$V_{signal}/CG =$ Number of Electrons
BER vs. Read Noise

\[ BER = \frac{1}{2} \text{erfc} \left( \frac{1}{\sqrt{8n_r}} \right) \]

What is an acceptable bit error rate?
BER vs. Read Noise

- Fossum 2011 WAG
- Fossum 2013
- Teranishi 2012

Bit Error Rate (BER)

Read Noise (e- rms)

1 / 3,000,000
1 / 2,500
1 / 20
Increased Dynamic Range

Sum of 16 fields
4@ T=1.0
4@ T=0.2
4@ T=0.04
4@ T=0.008

120 dB
Multi-bit Pixels

Counting low number of photoelectrons, e.g. 4b yields FW = 15 e-

Sum 4x4x16 = 256 pixels
Max = 15x256 = 3840

QIS: M=4096
4b: M=273

1b v. 4b
"Shot" Noise

\[ \sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 \]

plus

Read Noise (Gaussian model)

\[ P[k] = \frac{e^{-H} H^k}{k!} \]
Effect of Read Noise on Photoelectron Counting for Multi-bit Pixel

Note “peak” for H=5 is not at 5 e-
Summary

- Introduced concept of quanta exposure
- Quantified D-log H response
  - No adjustable parameters – should be very reproducible sensor to sensor.
  - Can retrieve H reliably over large range including non-linear portion (0.1 < H < 5)
- Quantified “shot” noise in QIS
- Effect of read noise on BER analyzed
- Extended DR with almost flat SNR discussed
- Multi-arrival and multi-bit pixels also addressed.
- More details in paper submitted to JEDS.