

# Read Noise Distribution Modeling for CMOS Image Sensors

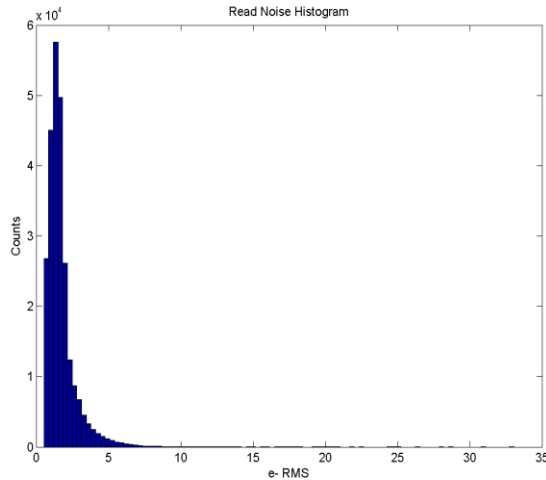
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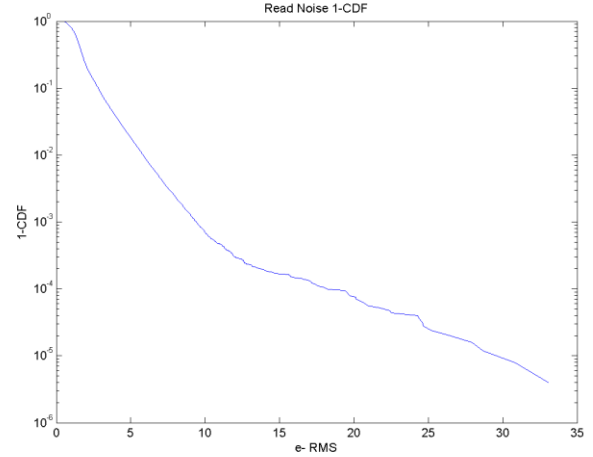
**Abstract**—We report on an empirical model that describes the read noise statistics in CMOS image sensors. This model includes pixel-to-pixel variations in thermal,  $1/f$  and RTS noise. It is based on the “unlucky pixel” concept where the source-follower FET channel is dominated by a single sub-channel and that RTS is due to the interactions of uniformly-distributed defects to this. In addition, we compare this model to measure data, and discuss how the read noise distribution is affected by various model parameters.

## I. INTRODUCTION

Temporal read noise is ideally dominated by thermal noise from the pixel source follower and is identically distributed (IID) for each pixel. However, when the readout noise is characterized in actual devices the noise power for each pixel is different. Figure 1 shows a typical read noise histogram from a CMOS image sensor, and Figure 2 shows the corresponding one minus the cumulative distribution function [1, 2]. As shown in Figures 1 and 2, when the input referred read noise of each pixel is plotted on a histogram a long tail is shown at the high end of the distribution. This is attributed to random telegraph signal (RTS) or other low-frequency noise sources.



**Figure 1: CMOS Image Sensor Read Noise Distribution**



**Figure 2: CMOS Image Sensor Read Noise 1-Cumulative Distribution Function**

The fundamental noise source in MOSFETs is Johnson noise caused by the thermal agitation of electrons in the conducting channel [3]. MOSFETs also have low frequency or “ $1/f$ ” noise, which grows as  $1/f^n$  as the frequency decreases. The  $1/f$  noise power spectrum for each MOSFET is different. These differences are caused by random variations in the fabrication process for each MOSFET. Correlated double sampling (CDS) is typically used in CMOS image sensors to high pass filter the  $1/f$  noise, and minimizes its impact on the sensor read noise. Unfortunately CDS does not completely mitigate the pixel-to-pixel read noise variations.

RTS is a special case of low-frequency noise that introduces a special problem when the amplitude is large and the frequency corresponds to the CDS period. The worst case occurs when the RTS variation changes between the reset sample and the signal sample, breaking the correlation used in the CDS and containing enough power to contribute significant temporal noise. This creates the noisiest pixels in the distribution tail. This is going to be explained by the “unlucky pixel” model described in the next section.

In this paper we develop an analytical model that can be used to predict the read noise tail distribution of CMOS image sensors. Our model is based on McWhorter’s number fluctuation theory of  $1/f$  noise in MOSFETs [3, 4]. We describe the functional distributions of the trap density, the trap lifetime, and the trap noise power consistent with observations based on extrinsic parameters (e.g., device

dimensions, doping profile) and intrinsic variables (e.g., defect densities). This will allow the use of this model to predict how process and design affect the read noise distribution of a CMOS image sensor.

In Section II we present the theory behind our model, in Section III we describe our model, in Section IV we present results from our model and compare to measure data. Finally in Section V we present summary conclusions.

## II. THEORY

For the traditional source-follower FET used in a pixel, we assume that charge is transported from the source to the drain, in the pixel source follower transistor, via the semiconductor surface at the oxide silicon interface. In addition, we assume that defects at the silicon-silicon dioxide interface can trap carriers. When the dimensions of the channel become small enough random dopant variations cause large variations in the surface potential of the source follower. We assume that this leads to many sub-channels, and in some cases to a single preferred sub-channel for carrier transport [5, 6]. This is analogous to an uneven stream bed with water flowing in it. The lowest areas of the stream bed are the preferred regions for water flow. The location of traps in the channel can cause very different noise characteristics in the MOSFET. If the trap is located in a region of the channel far away from a sub-channel then it can do little more than modulate the transistor threshold. On the other hand if it is located in a sub-channel is can significantly modulate the carrier flow. It is even worse if there is a dominant sub-channel with the trap located in the center of the sub-channel causing maximum variation in carrier flow.

When a trap is located in a sub-channel columbic repulsion causes a large variation in the carrier flow in the channel. This effect significant increases the low frequency noise of the MOSFET. Moreover, when a single dominant sub-channel interacts with a trap this can cause RTS noise. In this paper we will call this phenomenon an “unlucky” pixel.

To mitigate this effect either the process must be improved or the affect of the traps must be reduced. This is typically done by reducing the effect of the traps by moving the carrier transport below the surface of the silicon [8]. This effectively eliminates the sub-channel problem of tiny surface devices, but it also has many drawbacks. These include lower transconductance than surface channel devices, and lower gain than surface channel devices.

## III. NOISE MODEL

The goal of our noise model is to capture the statistics of “unlucky” pixels. Our noise model is stochastic in time  $t$  and in pixel location inside the array  $k$ . We assume that the noise of each pixel is wide sense stationary in time but not identically distributed in space. Figure 3 shows a block diagram of the pixel noise model. It assumes that all of the noise sources in the pixel can be input referred to the source follower.  $\eta_k$  is the input referred noise of pixel  $k$ ,  $S_{in,k}$  is the

input signal of pixel  $k$  and  $S_{out,k}$  is the output signal of pixel  $k$ . Due to the wide sense stationary assumption we can represent the input referred temporal noise power of each pixel  $k$  in the frequency domain as

$$\sigma_k^2 = c_{fd}^2 \int_0^\infty \left( \frac{4kT}{G_k} + \sum_{i=1}^{N_k} \frac{\gamma_{i,k} X_k^{\alpha_k}}{1 + (f / F_{i,k})^2} \right) H(f) df, \quad (1)$$

where

$$H(f) = \frac{|1 - e^{-j2\pi f\tau}|^2}{1 + (f / f_{LPF})^2}. \quad (2)$$

The first term in the integral is the thermal noise of the source follower, where  $G_k$  is the source follower trans-conductance of pixel  $k$ , and  $kT$  is Boltzmann’s constant times absolute temperature. The second term in the integral represents the excess low frequency noise of the source follower. The summation is over the number of traps in the channel  $N$ . Each trap in each pixel has an associated frequency  $F_{i,k}$  and a power  $\gamma_{i,k}$ . In addition, each pixel has an associated multiplier  $X_k^{\alpha_k}$ . This multiplier is used to differentiate between pixels with higher or lower excess low frequency noise.  $H(f)$  is the cascaded transfer functions of correlated double sampling (CDS) and low pass filtering.  $\tau$  is the sample period of the CDS function. This transfer function represents the signal processing that typically occurs in the column circuitry.

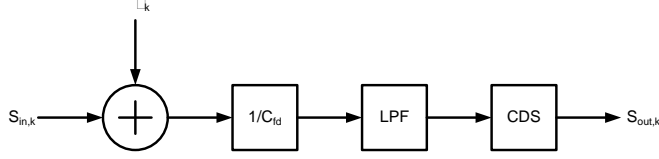
In this model  $G_k, N_k, F_{i,k}, X_k^{\alpha_k}$  are the random processes, indexed spatially, in the Monte Carlo simulation. We assume a long channel MOSFET transconductance model

$$G_k = \sqrt{2 \frac{W_k}{L_k} C_{ox} \mu_n I_k}, \quad (3)$$

where  $W_k$  is a white Gaussian random process with mean  $w$  and standard deviation  $dw$ , and  $L_k$  is a white Gaussian random process with mean  $l$  and standard deviation  $dl$ .  $C_{ox}$  is the gate oxide,  $\mu_n$  is the mobility of electrons in the channel, and  $I_k$  is a white Gaussian random process with mean  $i_{ds}$  and standard deviation  $di_{ds}$ .  $N_k$  is a white Poisson random process with parameter  $\beta W_k L_k$ .  $F_{i,k} = 10^{(U_{i,k} \log(f_{max} / f_{min}) + \log(f_{min}))}$ , where  $U_{i,k}$  is a white uniform random process defined between 0 and 1,  $f_{min}$  is the minimum trap frequency and  $f_{max}$  is the

maximum trap frequency. Note that  $\gamma_{i,k} = \frac{\chi \sqrt{f_{\min} f_{\max}}}{F_{i,k}}$ ,

where  $\chi$  is a constant.  $X_k$  is a white exponential random process with parameter 1, and  $\alpha_k = 2 + \frac{p_k}{2}$  where  $p_k$  is a Bernoulli random process, i.e.  $p_k$  takes on a value of either 1 or 0, with probability  $\delta$ . Note that when  $\alpha_k = 2.5$  the model is indicated a pixel with an “unlucky” source follower. In addition, the constants in  $\alpha_k$  could be modified to fit different semiconductor processes.



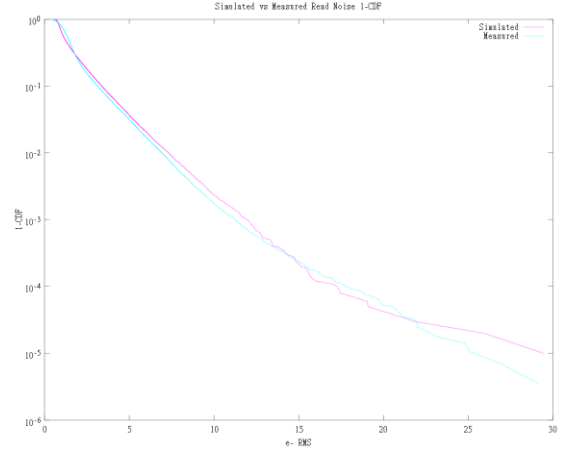
**Figure 3: Read Noise Model**

#### IV. SIMULATED RESULTS

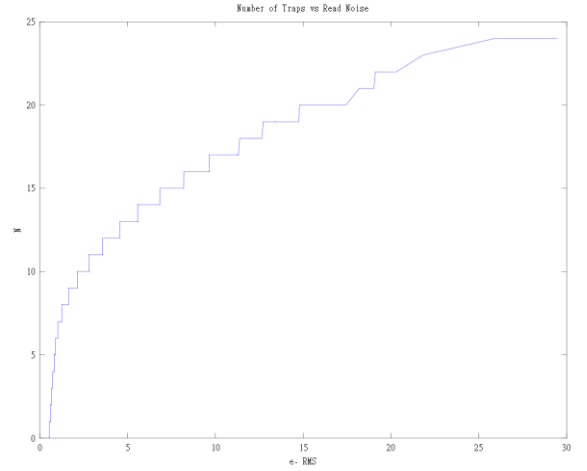
In this Section we present results from our model. First we show a comparison between measured data from a sCMOS image sensor [7] and our model. Then we change  $\alpha_k$  to a constant and show how this affects the read noise distribution. Finally we change  $X_k$  to a constant and show how this affects the read noise distribution.

Figure 4 shows measured data from a sCMOS image sensor and simulated data created by our model with the following parameters:

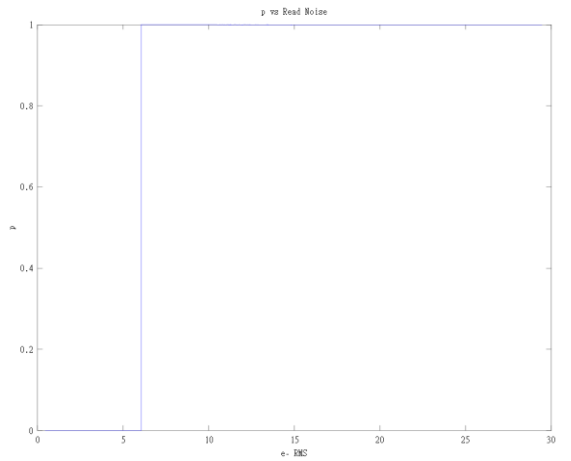
$c_{fd} = 5fF, dc_{fd} = 0.5fF, w = 1.5\mu m, \delta = 0.02,$   
 $dw = 0.05\mu m, l = 0.5\mu m, i_{ds} = 5\mu A, di_{ds} = 0.5\mu A,$   
 $\beta = 10, f_{\min} = 1Hz, f_{\max} = 10MHz, f_{LPF} = 1MHz,$   
 $\chi = 10^{-13}, C_{ox}\mu_n = 200\mu S, m = 100,$  and  $\tau = 10\mu s.$   
 Figures 5, 6, and 7 show the associated model parameters generated during the Monte Carlo simulation that created the data in Figure 3.



**Figure 4: Measured vs. Simulated Read Noise 1-CDF**



**Figure 5: N vs. Read Noise**



**Figure 6: p vs. Read Noise**

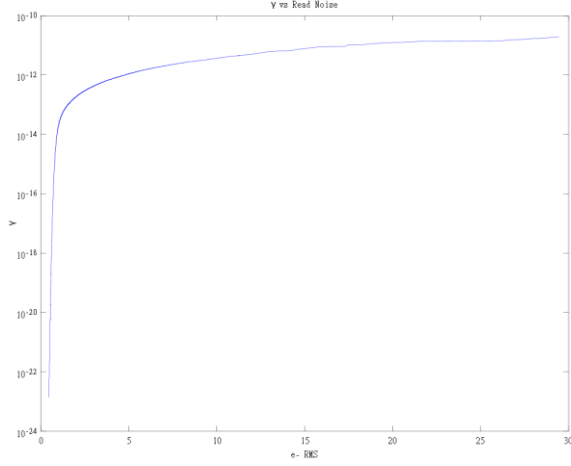


Figure 7: Gamma vs. Read Noise

In Figure 8 we compare the read noise distributions generated by our standard model to a model where  $\alpha_k = 2$  and then to a model where  $X_k = 1$ . As can be seen in Figure 8 forcing  $\alpha_k = 2$  causes the read noise distribution to have fewer very high noise pixels, and forcing  $X_k = 1$  causes the distribution to become very tight. In Table 1 we compare the median, the mean, and the RMS of the read noise distribution. Note that we define the RMS read noise over the sensor as the square root of the mean read noise squared plus the variance of the read noise distribution, i.e.  $\text{RMS read noise} = \sqrt{\mu_{\sigma_k}^2 + \sigma_{\sigma_k}^2}$ .

Clearly any mechanism that causes  $X_k = 1$  would allow us to build image sensors with a much more uniform read noise from pixel to pixel.

	Median	Mean	RMS
Standard	1.2e-	1.74e-	2.25e-
$\alpha_k = 2$	1.2e-	1.74e-	2.21e-
$X_k = 1$	1.6e-	1.62e-	1.67e-

Table 1: Read Noise Model Comparison

## V. DISCUSSION

Based on the "unlucky pixel" model, reducing RTS in surface-channel source-follower FETs is hard. It requires either smoothing out the channels or reducing dramatically the defect density related to the gate oxide.

An alternative proposed is a buried-channel source-follower FET. The limited available data is tantalizing in showing almost complete elimination of RTS noise [8] with a proposed explanation of the elimination of communication between the channel and the traps. The "unlucky pixel" model predicts this result due to the elimination of the sub-channels due to fringing fields.

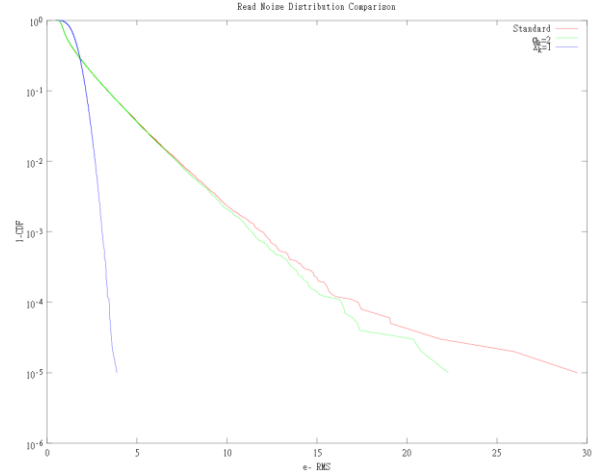


Figure 8: 1-CDF Comparison

## VI. CONCLUSIONS

In this paper we have described an empirical model for the read noise statistics in CMOS image sensors. This model includes variations in thermal, 1/f and RTS noise. It is based on the "unlucky pixel" concept where the source-follower FET channel is dominated by a single sub-channel and that RTS is due to the interactions of uniformly-distributed defects to this. In addition, we have compared this model to measure data, and discussed how the read noise distribution is affected by various model parameters.

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