Analysis of Reset Noise Suppression via Stochastic Differential Equations

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***Abstract*— Reset noise sets a fundamental detection limit on CMOS image sensors. Therefore, understanding the sources of reset noise and how to reduce them is essential for the design of low noise image sensors. In order to analyze reset noise suppression circuitry in CMOS image sensors we present a method for solving non-linear time varying stochastic differential equations with white noise inputs based on Itô calculus. This methodology can be used to obtain both closed form and numerical solutions to the cross correlation matrix for non-linear time varying stochastic systems. When a closed form solution is not available, this methodology can numerically calculate the transient noise power of a given circuit much faster than Monte Carlo based techniques. Therefore, this method can greatly speedup the noise optimization process during CMOS image sensor design.**

We analyze a reset noise suppression technique based on a 3T NMOS pixel. This technique reduces reset noise by using precise measurement and feedback control of the pixel voltage. We call this reset noise suppression technique "bandwidth control noise suppression." It uses a feedback loop to reduce reset noise by directly attenuating the in-band thermal noise of the reset transistor. If the noise bandwidth of the reset transistor is less than the bandwidth of the feedback loop, then the noise stored on the pixel after reset will be less than kTc. We present theoretical results for the noise suppression capability of this circuit, and compare them with both a numerical solution of the stochastic differential equations and a Monte Carlo simulation of the circuit.

I. INTRODUCTION

RESET noise sets a fundamental detection limit on CMOS image sensors. Therefore, understanding the sources of reset noise and how to reduce them is essential for the design of low noise image sensors. Recently many techniques have been presented for suppressing reset noise in CMOS image sensors [1]–[8]. Most of these methods rely on time varying and/or non–linear circuits that are difficult to analyze with traditional linear time invariant noise analysis techniques. The purpose of this paper is to present a theoretical framework for analyzing reset noise in time varying and/or non–linear systems, and use this framework to analyze a 3T NMOS pixel with reset noise suppression.

Currently most CMOS image sensor designers use SPICE based transient noise analysis to estimate the reset noise of their circuits. Typically these SPICE based solutions use Monte Carlo simulation to perform transient noise analysis. Monte Carlo simulation is computationally intensive, and therefore better suited to verification than to design. The noise analysis method presented in this paper is not based on Monte Carlo simulation. We present a method for solving time varying and/or non–linear stochastic differential equations with white noise inputs based on Itô calculus [9], [10]. This methodology can be used to obtain both closed form and numerical solutions to the cross correlation matrix for time varying and/or non-linear stochastic systems. When a closed form solution is not available, this methodology can numerically calculate the transient noise power of a given circuit much faster than Monte Carlo based techniques. Therefore, this method can greatly speedup the noise optimization process during CMOS image sensor design.

This paper is organized as follows: the next Section provides the notation and analysis framework for the remainder of the paper. The following Section presents a theoretical basis for our noise analysis, and describes a specific noise reduction method and circuit implementation. In Section IV we present simulation results for the circuit implementation presented in Section III and compare these results with theory. The final Section provides summary conclusions.

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II. RESET NOISE IN CAPACITIVE SENSORS

In this section we define the terms and assumptions for the remainder of the paper. We also discuss the voltage charge relationship for capacitive reset noise.

We assume that the thermal and shot noise sources that cause capacitive reset noise can be modeled as Gaussian white noise processes. We use upper case letters to denote random variables, and a random variable indexed by time denotes a random process, i.e. $X(t)$. A specific value of a random variable is denoted by a lower case letter, i.e. x represents a specific value of the random variable X. The cross correlation function of two random processes at any time t is defined as

$$\sigma^2_{XY}(t) = E[X(t)Y(t)] - E[X(t)]E[Y(t)], \qquad (1)$$

where $E[XY] = \int_{-\infty}^{\infty} xyf(x,y)dx$, and $f(x,y)$ is the joint distribution of X and Y. If either $X(t)$ or $Y(t)$ is a zero mean process then equation 1 simplifies to

$$\sigma^2_{XY}(t) = E[X(t)Y(t)]. \qquad (2)$$

The variance, i.e. power, of a zero mean random process $X(t)$ at time t is defined as the cross correlation of the process with itself,

$$\sigma^2_X(t) = E[X(t)X(t)]. \qquad (3)$$

If $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ are vectors of random processes, then the cross correlation matrix is

$$\Sigma_{\mathbf{XY}}(t) = E[\mathbf{X}(t)\mathbf{Y}(t)^T] - E[\mathbf{X}(t)]E[\mathbf{Y}(t)]^T, \quad (4)$$

and the autocorrelation matrix is

$$\Sigma_{\mathbf{X}}(t) = E[\mathbf{X}(t)\mathbf{X}(t)^T] - E[\mathbf{X}(t)]E[\mathbf{X}(t)]^T. \quad (5)$$

We define $\mathbf{W}(t)$ as a vector of independent identically distributed Weiner processes [11]. Note that each Wiener process is normally distributed with zero mean and variance t $\forall$ $0 \leq t < \infty$.

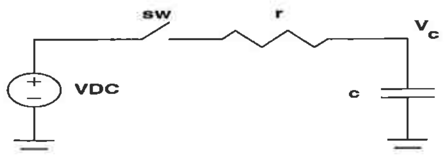

Fig. 1. RC Reset Circuit

When a capacitor, c, is reset to a DC voltage through a resistive switch, as shown in Figure 1, and the switch is abruptly opened the noise voltage power on the capacitor is $\sigma^2_{V_c} = \frac{kT}{c}$ [12]. Note that the voltage noise power is independent of r and the final DC voltage across the capacitor. Through the remainder of this paper we will call this type of capacitive reset "hard reset." Since voltage times capacitance equals charge, the charge noise stored on the capacitor after reset is $\sigma^2_{Q_c} = kTc$. The RMS charge noise in electrons is $\frac{\sqrt{kTc}}{q}$, where q is the charge of an electron. This shows that larger capacitors reduce the voltage noise, while on the contrary smaller capacitors reduce the charge noise.

III. Reset Noise Analysis

In this Section we present a method for solving linear time varying stochastic differential equations (SDEs) based on Itô calculus [9], [10]. This method can also be extended to non–linear time varying SDEs. A complete description of this method applied to circuit simulation is given by Demir [13].

Using this method for solving SDEs we analyze the reset noise performance of a pixel first described by Loose et al. in [4]. We call the reset noise reduction technique used by this pixel "bandwidth control." This technique uses a feedback loop to directly attenuate the thermal noise of the reset transistor. Moreover, if the noise bandwidth of the reset transistor is less than the bandwidth of the feedback loop, then the noise voltage stored on the capacitive sensor can be reduced below $\frac{kT}{c}$.

Reducing reset noise using bandwidth control is a two step process. First, hard reset is performed on the capacitive sensor. Then an error amplifier in a feedback loop is connected via a time varying resistor to the capacitive sensor. Finally, the resistance of time varying resistor is increased until the bandwidth of the error amplifier is much larger than the noise bandwidth of the resistor.[1] In addition, $r(t) \to \infty$ as $t \to \infty$, i.e. the resistor that connects the error amplifier to the capacitive sensor must become disconnected by the end of the reset cycle.

[1]If the resistor's noise is to be attenuated, the bandwidth of the error amplifier must be larger than the noise bandwidth of the time varying resistor for a significant period of the reset cycle.

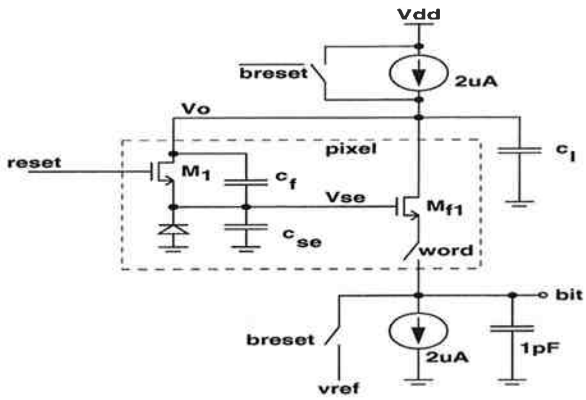

Fig. 2. Bandwidth Control Schematic

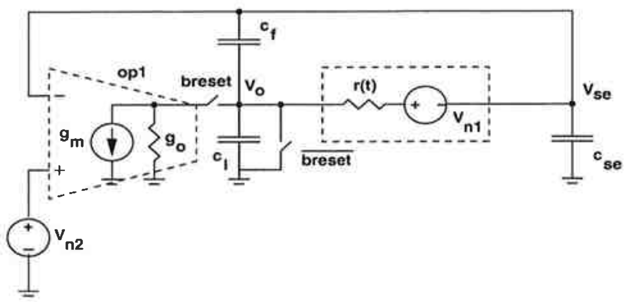

Fig. 3. Simplified Bandwidth Control Model

A circuit that implements bandwidth control is shown in Figure 2. A simplified linear, but time varying, model of this circuit is shown in Figure 3, and the timing waveforms for the model are shown in Figure 4. After hard reset is complete,

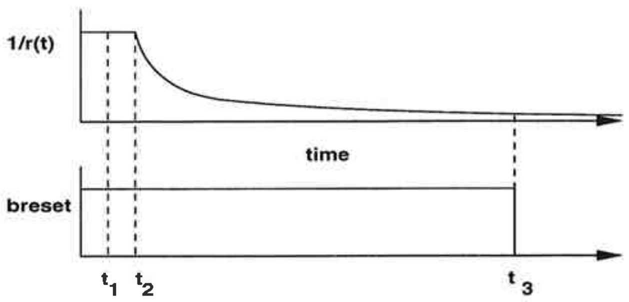

Fig. 4. Bandwidth Control Timing Waveforms

i.e. $t_1 < t < t_2$, the capacitive sensor noise voltage power is approximately equal to $\frac{kT}{(c_{se}+c_f)}$. When $r(t)$ is increased after t_2 the voltage noise, $\sigma^2_{V_{se}}(t)$, on the capacitive sensor can be determined by finding the covariance matrix of

$$\mathbf{V}(t) = \begin{bmatrix} V_o(t) \\ V_{se}(t) \end{bmatrix}. \quad (6)$$

$\mathbf{V}(t)$ satisfies the following set of SDEs

$$\mathbf{A}\frac{d\mathbf{V}(t)}{dt} = \mathbf{B}\mathbf{V}(t) + \mathbf{C}\frac{d\mathbf{W}(t)}{dt} \tag{7}$$

where

$$\mathbf{A} = \begin{bmatrix} c_l + c_f & -c_f \\ -c_f & c_{se} + c_f \end{bmatrix}, \tag{8}$$

$$\mathbf{B} = \begin{bmatrix} -\frac{1}{r(t)} - g_o & \frac{1}{r(t)} - g_m \\ \frac{1}{r(t)} & -\frac{1}{r(t)} \end{bmatrix}, \tag{9}$$

and

$$\mathbf{C} = \begin{bmatrix} \sqrt{\frac{2kT}{r(t)}} & \sqrt{2kT\gamma g_m} \\ -\sqrt{\frac{2kT}{r(t)}} & 0 \end{bmatrix}. \tag{10}$$

To simplify our notation, let

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B}, \tag{11}$$

and

$$\mathbf{E} = \mathbf{A}^{-1}\mathbf{C}. \tag{12}$$

Assuming the initial condition

$$E[\mathbf{V}(0)] = \mathbf{0}, \tag{13}$$

then

$$E[\mathbf{V}(t)] = \mathbf{0} \;\; \forall \;\; t \geq 0. \tag{14}$$

Since $E[\mathbf{V}(t)] = \mathbf{0}$, $\mathbf{\Sigma_V}(t) = E[\mathbf{V}(t)\mathbf{V}(t)^T]$, it can be shown that $\mathbf{V}(t)$ is an Itô process [9], [10], and therefore using the Itô formula [9] for stochastic differentials we find that

$$\begin{aligned} d\mathbf{V}(t)\mathbf{V}(t)^T &= (\mathbf{D}\mathbf{V}(t)\mathbf{V}(t)^T + \mathbf{V}(t)\mathbf{V}(t)^T\mathbf{D}^T \\ &\quad + \mathbf{E}\mathbf{E}^T)dt + \mathbf{V}(t)(\mathbf{E}d\mathbf{W}(t))^T \\ &\quad + (\mathbf{E}d\mathbf{W}(t))\mathbf{V}(t)^T. \end{aligned} \tag{15}$$

Taking the expectation of both sides of equation 15 and dividing by dt we find the following set of deterministic differential equations

$$\frac{d\mathbf{\Sigma_V}(t)}{dt} = \mathbf{D}\mathbf{\Sigma_V}(t) + \mathbf{\Sigma_V}(t)\mathbf{D}^T + \mathbf{E}\mathbf{E}^T. \tag{16}$$

Using equation 16 with the initial condition

$$\mathbf{\Sigma_V}(0) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{kT}{c_{se}+c_f} \end{bmatrix} \tag{17}$$

we can numerically determine $\sigma^2_{V_{se}}(t)$, $\sigma^2_{V_o}(t)$, and $\sigma^2_{V_{se}V_o}$. In order to derive a closed form result we let $r(t)$ be a constant, r, and then it can be shown that as $t \rightarrow \infty$

$$\begin{aligned} \sigma^2_{V_{se}} &= \tfrac{kT}{\beta}(c_l c_{se} r g_o^2 + c_f c_{se} r g_o^2 + c_f c_l r g_o^2 + \\ &\quad c_f^2 \gamma g_m r g_o + c_l^2 g_o + c_f^2 \gamma g_m^2 r + c_l c_{se} \gamma g_m + \\ &\quad c_f c_{se} \gamma g_m + c_f c_l \gamma g_m + c_l^2 g_m), \end{aligned} \tag{18}$$

$$\begin{aligned} \sigma^2_{V_o} &= \tfrac{kT}{\beta}(c_{se}^2 \gamma g_m r g_o + 2c_f c_{se} \gamma g_m r g_o + \\ &\quad c_f^2 \gamma g_m r g_o + c_{se}^2 g_o + c_{se}^2 \gamma g_m r + \\ &\quad 2c_f c_{se} \gamma g_m^2 r + c_f^2 \gamma g_m^2 r + c_l c_{se} g_m^2 r + \\ &\quad c_f c_{se} g_m^2 r + c_f c_l g_m^2 r + c_l c_{se} \gamma g_m + \\ &\quad c_f c_{se} \gamma g_m + c_f c_l \gamma g_m + c_{se}^2 g_m), \end{aligned} \tag{19}$$

and

$$\begin{aligned} \sigma^2_{V_{se}V_o} &= \tfrac{kT}{\beta}(c_f c_{se} \gamma g_m r g_o + c_f^2 \gamma g_m r g_o - \\ &\quad c_l c_{se} g_m r g_o - c_f c_{se} g_m r g_o - c_f c_l g_m r g_o - \\ &\quad c_l c_{se} g_o + c_f c_{se} \gamma g_m^2 r + c_f^2 \gamma g_m^2 r + \\ &\quad c_l c_{se} \gamma g_m + c_f c_{se} \gamma g_m + c_f c_l \gamma g_m - \\ &\quad c_l c_{se} g_m) \end{aligned} \tag{20}$$

where

$$\begin{aligned} \beta &= ((c_l c_{se} + c_f c_{se} + c_f c_l)(g_o + g_m) \\ &\quad (c_{se} r g_m + c_f r g_o + c_f g_m r + c_{se} + c_l)). \end{aligned} \tag{21}$$

For example, if we assume that $t_3 - t_2$ is sufficiently long and that $T = 300$K, $c_l = 0.3$pF, $\gamma = 1$, $c_f = 0.4$fF, $c_{se} = 6$fF, $g_m = 21.3\mu$S, $g_o = 358$nS, and $r = 400\text{M}\Omega$ then $\sigma^2_{V_{se}}(t_3) = 29\text{n}V^2$, $\sigma^2_{V_o}(t_3) = 8.7\mu\text{V}^2$, and $\sigma^2_{V_{se}V_o}(t_3) = -72.6\text{nV}^2$. Note that r was selected such that $2\pi r(c_{se} + c_f) = 16\mu$s. If we numerically solve equation 16 for $0 \leq t \leq 20\mu$s using $r(t) = r_0 \exp(\frac{t}{\tau})$, where $r_0 = 10\text{K}\Omega$ and $\tau = 1\mu$s, then $\sigma^2_{V_{se}}(t_3) = 3.2\text{n}V^2$, $\sigma^2_{V_o}(t_3) = 8.9\mu\text{V}^2$, and $\sigma^2_{V_{se}V_o}(t_3) = -0.137\mu\text{V}^2$. Figure 5 shows the numerical solutions of equation 16 for $0 \leq t \leq 20\mu$s with five different values of τ ranging from 0.25μs to 4μs and $r_0 = 10\text{K}\Omega$. As expected

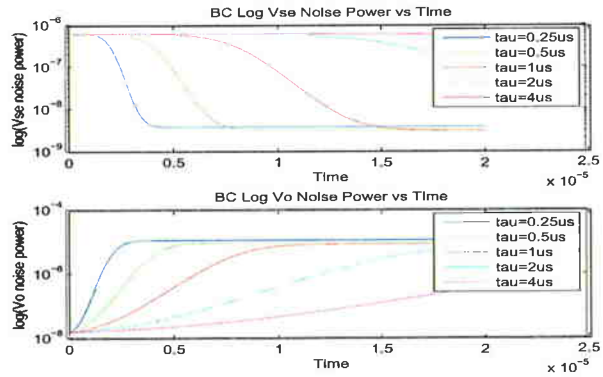

Fig. 5. Numerical Solutions of Bandwidth Control SDEs

the closed form approximation and the numerical solution are not very close due to the time varying nature of the SDEs. After t_3 the topology of the circuit is modified for readout by closing/opening the switches controlled by *breset*. This causes noise stored on c_l to be transferred onto c_{se} via c_f. By considering conservation of charge for c_{se} and c_f, it can be shown that after t_3 the noise voltage on c_{se} is

$$\begin{aligned} \sigma^2_{V_{se}}(t > t_3) &= \tfrac{1}{(c_f+c_{se})^2}(2c_f c_{se}(\sigma^2_{V_{se}}(t_3) - \\ &\quad \sigma^2_{V_{se}V_o}(t_3)) + \\ &\quad c_f^2(\sigma^2_{V_{se}}(t_3) + \sigma^2_{V_o}(t_3) - \\ &\quad 2\sigma^2_{V_{se}V_o}(t_3)) + c_{se}^2 \sigma^2_{V_{se}}(t_3)). \end{aligned} \tag{22}$$

Using the results from the closed form solution $\sigma^2_{V_{se}}(t > t_3) = 73\text{n}V^2$, and $\frac{\sigma_{q_{se}}}{q}(t > t_3) = 10.8$e- RMS, and using the results from the numerical solution $\sigma^2_{V_{se}}(t > t_3) = 55\text{n}V^2$, and $\frac{\sigma_{q_{se}}}{q}(t > t_3) = 9.4$e- RMS. Note that noise injected by c_f onto c_{se} at $t = t_3$ dominates the readnoise. Therefore, minimizing c_f is critical for low noise reset and readout of c_{se}.

IV. Simulation

In this section we present circuit simulations that demonstrate the reset noise reduction technique described in Section III. The simulation results were generated using HPSPICE's transient noise analysis tool. HPSPICE uses Monte Carlo simulation for transient noise analysis. BSIM3 level 3 SPICE models, from a 0.18μm CMOS process, were used for the simulation. The simulations were performed using 512 runs with $T = 25°$C, $V_{dd} = 2.8$V, and $V_{reset} = 2.0$V.

Figure 6 shows the SPICE input waveforms used for simulating the bandwidth control circuit, shown in Figure 2, and the SPICE generated output waveforms. Table I shows all of the

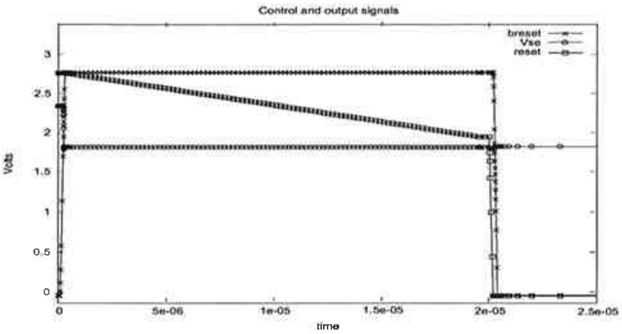

Fig. 6. Bandwidth Control Input/Output Waveforms

capacitor, transistor and amplifier characteristics for Figure 2. Figure 7 shows the transient noise simulation of the bandwidth

Component	Parameter	Value
c_l	capacitance	0.3pF
c_f	capacitance	0.4fF
c_{se}	capacitance	6fF
M_1	W/L	0.42μm / 0.6μm
M_1	threshold voltage	0.45V
M_{f1}	W/L	1.2μm / 0.5μm
M_{f1}	threshold voltage	0.45V
M_{f1}	transconductance	36.2μS
M_{f1}	body effect conductance	8.3μS
M_{f1}	output conductance	360nS
$vref$	voltage	1.0V

TABLE I
BANDWIDTH CONTROL SIMULATION COMPONENT PARAMETERS

control circuit. The reset noise voltage power on the capacitive sensor $\sigma^2_{V_{se}}$ at 5μs is approximately $0.4\mu\text{V}^2$. This result corresponds to about 60% of the predicted hard reset value $\frac{kT}{c_{se}+c_f}$. The noise power $\sigma^2_{V_{se}}$ after 25μs is 68nV^2. Therefore the reset noise reduction factor, RNRF $= \frac{\sigma^2_{V_{se}}}{kT/(c_{se}+c_f)}$, is 9.5. Theoretical and simulated noise results are compared in Table II. The Table column headings are: CFS is closed form result, NS is numerical solution, and MCS is Monte Carlo simulation. Using the same computer, the time required to solve the SDEs numerically (with Matlab) is approximately 15 seconds, and the time required to perform the HPSPICE Monte Carlo simulation is approximately 15 minutes.

V. Conclusion

We have presented a theoretical framework for understanding and modeling reset noise suppression in CMOS image

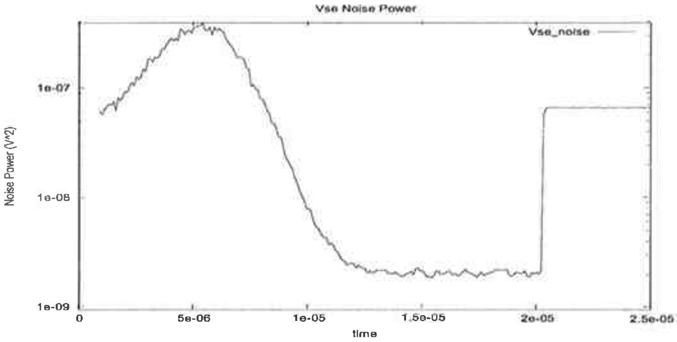

Fig. 7. Bandwidth Control Transient Noise Waveforms

	CFS	NS	MCS
$\sigma^2_{V_{se}}(t = 25\mu s)$	73nV^2	55nV^2	68nV^2
RNRF	8.9	11.8	9.5

TABLE II
SUMMARY OF RESULTS

sensors. This method has been demonstrated on a circuit which is currently in use. We also showed that simulated results correspond closely with theory.

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