P15 Non-linear AD conversion, tolerant for pixel offset errors

Bart Dierickx
IMEC, Kapeldreef 75, B-3001 Leuven, Belgium
Dierickx@imec.be

The response function of a pixel typically exhibits offset and gain non-uniformity. If the overall sensor response, from pixel signal S to digital output D, is perfectly linear, these errors can easily be corrected by off-chip additions and multiplications. We prove that, if the overall sensor response is non-linear, this is not possible unless the non-linearity follows the conversion law $D = \alpha - exp(\beta S)$. We compare this law with the gamma conversion and other non-linear laws. A simple implementation of this law in a flash AD converter is proposed.

1. Non linear conversion

It is in many cases desirable that the voltage response or the digital response of an image sensor is a highly linear function of the impinging light power. This allows for straightforward pixel offset and gain correction, by addition or multiplication. If this is done in the digital domain, the AD-conversion should be linear.

Suppose that *D(S)* is the AD conversion function of the optical signal (voltage) *S*. If a pixel suffers from an additive error *A*, this must be corrected by an *offset* subtraction afterwards in order to obtain the correct value. Then

$$D(S) = D(S+A) - offset \qquad (1)$$

In the above case, the only solution for D(S) is a linear function.
Suppose that we have the possibility to correct with an offset and a gain. What is the general solution for D(S)?

$$D(S) = D(S+A) * gain - offset \qquad (2)$$

It is easy to demonstrate that the general solution of D(S) should obey

$$D'(S) / D'(S+A) = constant, \qquad (3)$$

or that the derivative of D(S) is an exponential function, thus:

$$D(S) = \alpha +/- exp(\beta S) \qquad (4)$$

In practice D(S) represents the conversion law of an AD converter, projecting the useful signal (S) range on the digital output range (D) of the ADC. If both input and output ranges are normalized to the interval [0..1], the function is written as:

$$D(S) = 1 - exp(\beta S)/exp(\beta) \qquad (5)$$

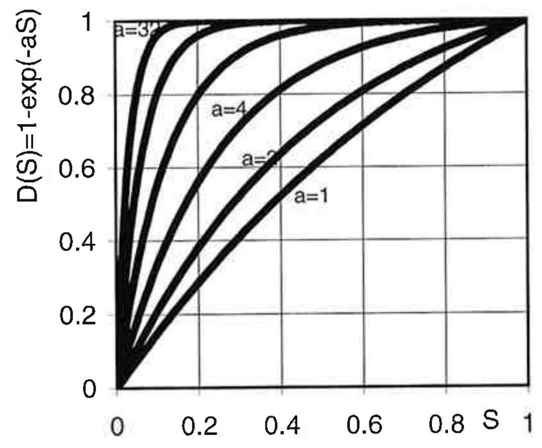


Fig. 1. Set of responses D(S)=1-exp(-aS)/exp(-a), a varying between 1 and 32

2. Other deliberately non-linear responses

In many practical cases image sensors, camera systems and film deliberately exhibit non-linear responses. The reasons for this are

2.1. Matching the sensor noise

In order to use the output dynamic range of a sensor in the most optimal way, the noise should have a constant amplitude, seen on the final output scale F(S), where S is the optical signal, and F is the final sensor output.

$$dF(S)/dS * N(S) = \text{constant} \qquad (6)$$

•If the noise of the pixels does not depend on the light level, the linear response is the most appropriate.

•If the noise amplitude obeys a square-root law, as for optical shot noise,

$$N(S) = \text{sqrt}(\alpha S) \qquad (7)$$

And thus the best F(S) is

$$F(S) = (\alpha S)^{0.5} + \beta \qquad (8)$$

Where α and β are constants.

•If the noise amplitude is linearly proportional to the signal, as for PRNU, or if the image quality is limited by relative contrast differences, we have another situation:

$$N(S) = \alpha S \qquad (9)$$

$$dF(S)/dS = \alpha S \qquad (10)$$

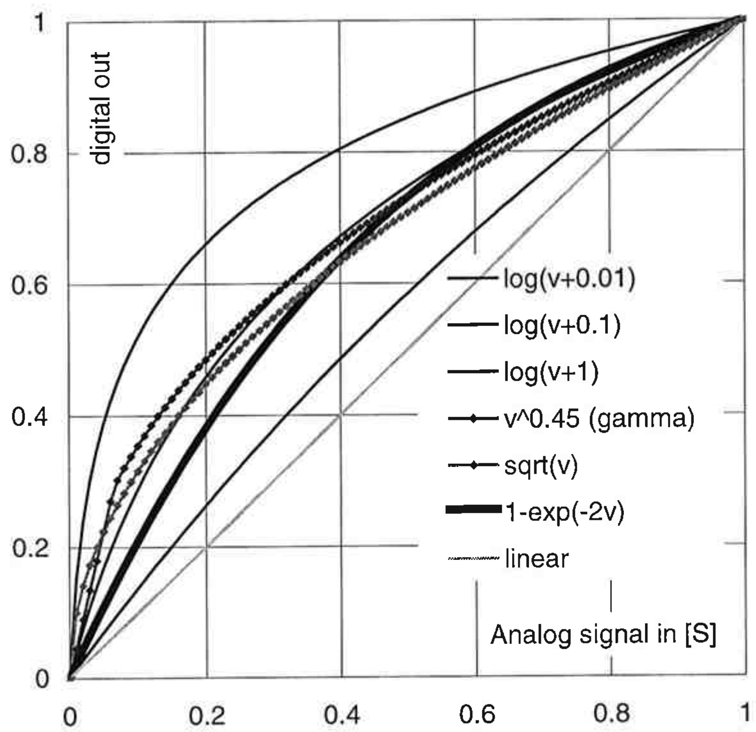
an the ideal overall response is logarithmic

$$F(S) = \log(\alpha S) + \beta \qquad (11)$$

2.2. The gamma-correction

The gamma correction function was originally introduced in television cameras as an attempt to compensate for the non-linear optical behavior of the CRT display device.

$$F(S) = S^{0.45} \qquad (12)$$



The exponent in the function may vary, but commonly the gamma function is used with an exponent ("gamma") of about *0.45.* Near S=0 the gamma function is limited to a linear response F(S)<4.5*S.

Although the original reason for the introduction of the gamma correction is archaic, it still is a near ideal response for shot noise limited sensors. At normal illumination levels it is a root function; in the dark it is a linear function matched to a base level noise that is independent from the signal amplitude.

fig. 2 Comparison of several non-linear conversion laws: linear response and examples of the logarithmic, square root, gamma and the present non-linear law.

3. Correction for both

offset and gain errors

Besides an offset error A, the pixel signal S can have a gain error B, and even higher order errors. What is the general solution for D(S), if it should be only gain and offset corrected.

$$D(S) = (D(B*S+A)) * gain - offset \qquad (13)$$

It is clear that the general solution of formula (4) is not valid when *B* is different from 1. However, as the application is the correction of PRNU, where B is close to 1, (4) can be a good approximation

$$D(S) = \alpha - \exp(\beta S) \qquad (14)$$

$$D(BS+A)) = \alpha - \exp(\beta BS+\beta A) = \alpha - \exp(\beta BS) * \exp(\beta A) \qquad (15)$$

$$(D(B*S+A)) * gain - offset = [gain*\alpha - offset] - [gain*\exp(\beta A)] * \exp(\beta BS) \qquad (16)$$

As B is close to 1, we

$$(D(B*S+A))*gain-offset = [gain*\alpha-offset] - [gain*\exp(\beta A)]*\exp(\beta S)*\exp(\beta S(B-1)) \qquad (17)$$

The argument of $\exp(\beta S(B-1))$ is small, thus we can approximate it with $1+\beta S(B-1)$, transforming (17) in

$$(D(B*S+A))*gain-offset = [gain*\alpha-offset] - [gain*\exp(\beta A)*(1+\beta S(B-1))] * \exp(\beta S) \qquad (18)$$

which can be used to solve (13):

$$\alpha - \exp(\beta S) = [gain*\alpha-offset] - [gain*\exp(\beta A)*(1+\beta S(B-1))] * \exp(\beta S) \qquad (19)$$

4. Implementation in a flash ADC

By varying the values of the resistors between two comparator stages of a flash ADC, in principle any monotonous non linear law can be obtained. The comparator levels for the function

$$D(S) = \alpha - \exp(\beta S)$$

should obey

$$S(D) = \log(D-\alpha) / \beta$$

The resistor between two consecutive comparators (between D = n and D = n+1) is

$R(D) = dS(D)/dD$, thus:

$R(D) = 1/(\beta(D-\alpha))$, or written as conductances:

$$G(D) = \beta(D-\alpha)$$

The conductance is a simple linear function of the ADC code. Such a resistor ladder is straighforward to implement as a triangular sheet of polysilicon:

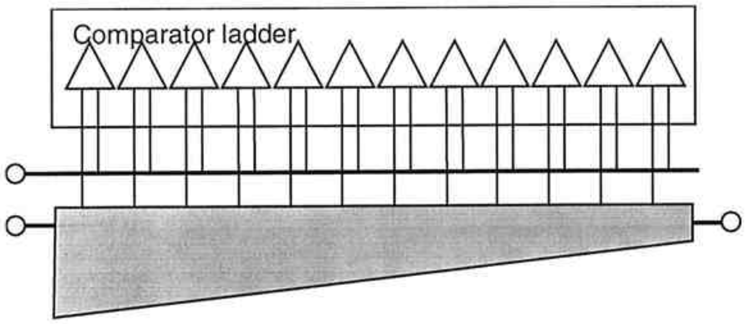


fig. 3 schematic of the non-linear resistor chain in a flash ADC. The conductance between tap pairs varies in a linear way.