

DESIGN OF SOLID-STATE IMAGERS

MARVIN H. WHITE

Sherman Fairchild Center
Lehigh University
Bethlehem, Penn., U.S.A.

OUTLINE

- **Introduction**

 - Photo Sensor Array
 - Noise
 - Bandwidth
 - Figure-of-Merit
 - Photo Sensor Array Design
 - Conclusions

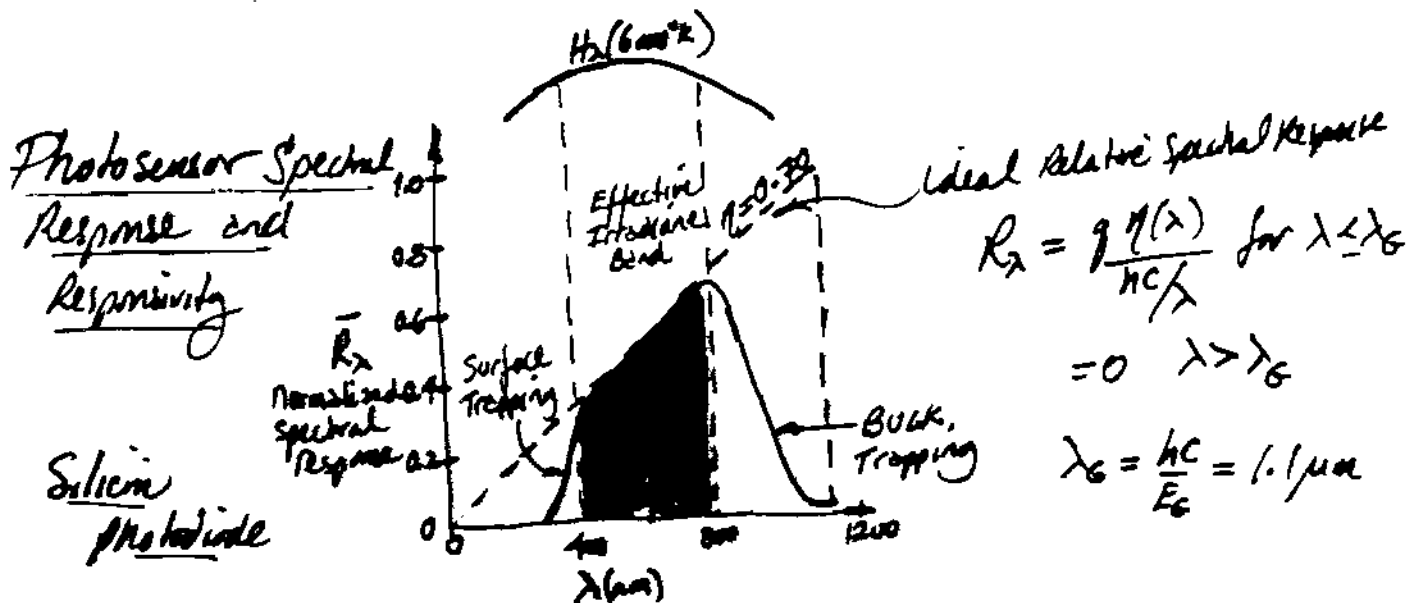
- **Noise Considerations**

- **Bandwidth Limitations**

- **Figure-of-Merit Formulation**

- **Photo Sensor Array Design**

- **Conclusions**



$$\eta \approx 1 - \left(\frac{R_{Si} - R_{air}}{R_{Si} + R_{air}} \right)^2 = 1 - \left(\frac{3.45 - 1}{3.45 + 1} \right)^2 \approx 0.70$$

Quantum efficiency is almost constant in the 'effective' irradiance band.

Spectral Response

$$R_\lambda = \frac{q \eta(\lambda)}{hc/\lambda} = \frac{J_\lambda}{H_\lambda} \left(\frac{A}{W} \right)$$

Current density output per irradiance input

Irradiance

$$H_\lambda = \frac{2\pi c^2 h}{\lambda^5 (e^{hc/\lambda kT_s} - 1)} \left(\frac{W}{m^2 \cdot \mu m} \right)$$

Ideal Planckian Blackbody Source

Responsivity

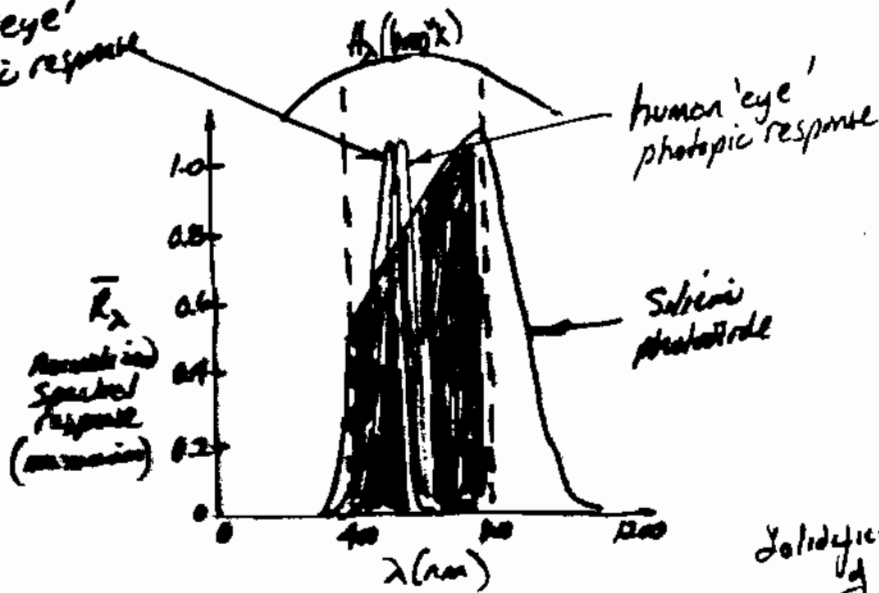
$$R = \frac{J}{H_{eff}} = \frac{\int_{\lambda_1}^{\lambda_2} d\lambda R_\lambda H_\lambda}{\int_{\lambda_1}^{\lambda_2} d\lambda H_\lambda}$$

Let $x = hc/\lambda kT_s$, $\lambda_2 = 800 \text{ nm}$, $\lambda_1 = 400 \text{ nm}$, $T_s = 6000^\circ K$, $\eta = 0.7$

$$R = \frac{q \eta}{kT_s} \frac{\int_{x_1}^{x_2} dx x^2 e^{-x}}{\int_{x_1}^{x_2} dx x^3 e^{-x}} = 0.33 \frac{A}{W}$$

Radiometric and Photometric Considerations

human 'eye' scotopic response



Definition of λ : $680 \text{ lm} = 680 \text{ W} \cdot 10^{-4} \frac{\text{lumens}}{\text{watt}} = R_{\lambda}(\text{max}) \int_0^{\infty} \bar{R}_{\lambda}(\text{eye}) H_{\lambda}(2042^{\circ}\text{K}) d\lambda$

note: 1 meter-candle = $1 \frac{\text{lumen}}{\text{m}^2} = 1 \text{ lux}$
 $= 10 \frac{\text{lumen}}{\text{ft}^2} = 10 \text{ ft-candles}$

680 lumens/watt at the spectral peak ($\lambda_m = 556 \text{ nm}$) for the photopic Eye Response

$$F (\text{luminance}) = 680 \int_0^{\infty} \bar{R}_{\lambda}(\text{eye}) H_{\lambda}(T_T) d\lambda$$

$$K (\text{luminosity}) = \frac{F}{\int_0^{\infty} f(\lambda) H_{\lambda}(T_T) d\lambda} = \frac{680 \int_0^{\infty} \bar{R}_{\lambda}(\text{eye}) H_{\lambda}(T_T) d\lambda}{\int_0^{\infty} f(\lambda) H_{\lambda}(T_T) d\lambda} \left(\frac{\text{lumens}}{\text{watt}} \right)$$

cut-off of optical transmission by glasses

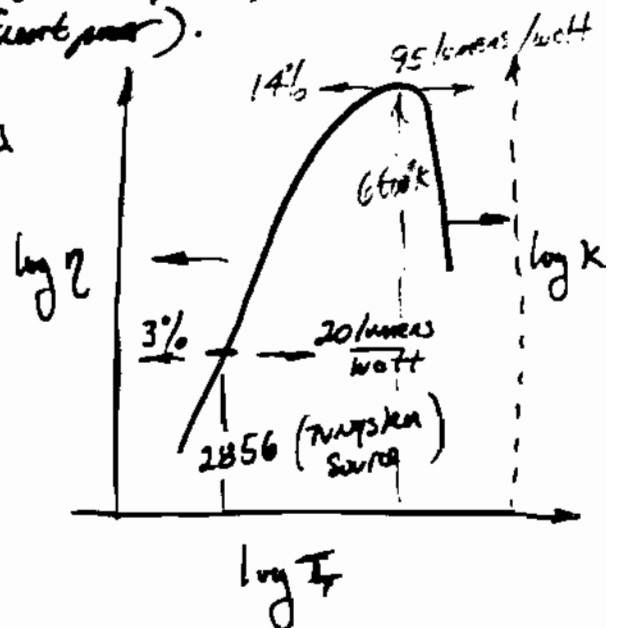
The luminosity is the ability of the test source to convert watts (radiant power) into lumens (luminous power).

$$\eta (\text{efficiency}) = 100 \frac{\int_0^{\infty} \bar{R}_{\lambda}(\text{eye}) H_{\lambda}(T_T) d\lambda}{\int_0^{\infty} f(\lambda) H_{\lambda}(T_T) d\lambda}$$

$T_T = 6000^{\circ}\text{K}$
 $\lambda_2 = 800 \text{ nm}$
 $\lambda_1 = 400 \text{ nm}$
 $K_S = 200 \frac{\text{lumens}}{\text{watt}}$

Photo sensor Sensitivity

$$S = \frac{R}{K_S} = \frac{0.37 \text{ A/W}}{200 (\text{lm/W})} = \frac{1.65 \text{ mA}}{\text{lm}}$$



Detector Responsivity

$$R_D = R A_D$$

example. $A_D = 15 \mu\text{m} \times 20 \mu\text{m}$

$$R_D = \frac{0.33 \text{ A}}{\text{W}} (15 \times 20 \mu\text{m}^2)$$

$$= \frac{0.099 \mu\text{A}}{\mu\text{J}/\text{m}^2} \left(\frac{616 \text{ e}^-}{\mu\text{J}/\text{m}^2} \right)$$

Signal charge

$$Q_S = R_D \underbrace{I_{ph}}_{\text{photon generated current}} \tau = R_D \underbrace{E_{eff}}_{\text{exposure density } (\mu\text{J}/\text{m}^2)}$$

$\xrightarrow{\text{exposure time}}$

$\xrightarrow{\text{detector responsivity } (\text{pC}/\mu\text{J}/\text{m}^2)}$

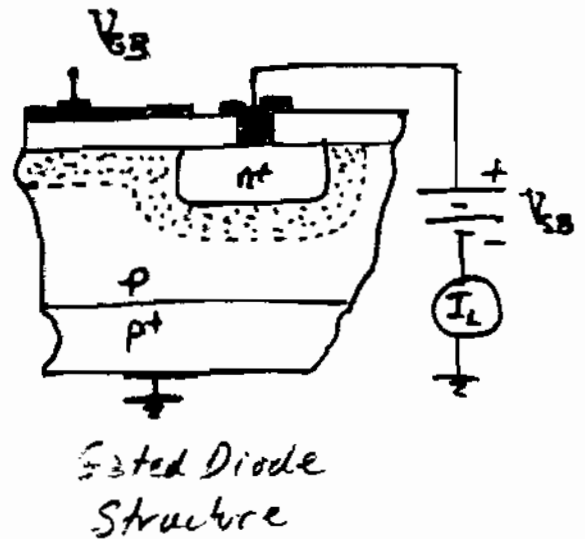
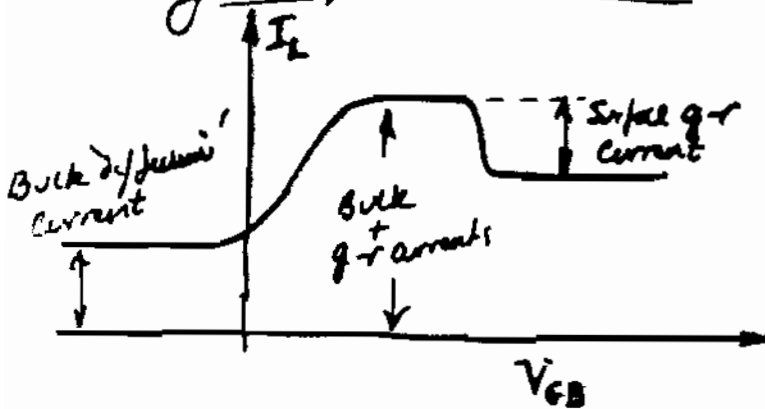
In general, we have the leakage current which adds to the signal.

$$Q_S = (I_{ph} + I_L) \tau$$

where,

$$I_L = \underbrace{\frac{q A_i W_j}{\tau_{gr}} A_D}_{\text{g-r current in bulk depletion region}} + \underbrace{q A_i S_0 A_S}_{\text{g-r current at the surface}} + \underbrace{q \frac{A_i^2}{N_B} \sqrt{\frac{D_n A_D}{\tau_n}}}_{\text{bulk 'diffusion' current}}$$

Test Structure for Determination of Leakage Current Components



NOISE SOURCES

- **Internal**

- (a) **Random Noise (Time-Variant)**

- Thermal or 'Nyquist' Noise
 - Shot or 'Schottky' Noise
 - **Generation-Recombination** or '1/f' Noise

- (b) **Fixed Pattern Noise (Time-Invariant)**

- Temperature
 - Responsivity
 - Spectral Response

- **External**

- Signal Shot Noise
 - Quantization (A/D Converter) Noise
 - Preamplifier (OPAMP) Noise
 - Power Supply Noise
 - Radiation 'Interference' Noise

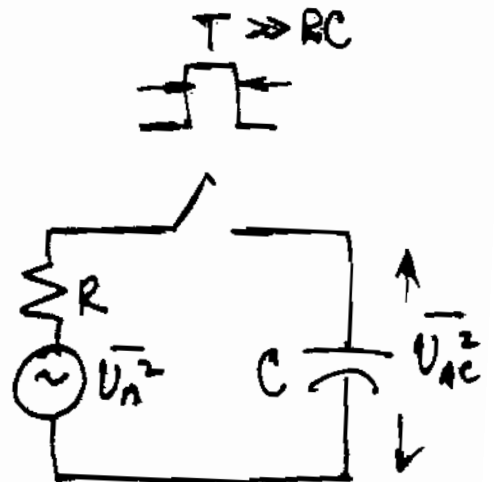
Nyquist 'Thermal' Noise

Johnson Nyquist Noise

$$\overline{V_n^2} = 4kTR \Delta f$$

$$\overline{V_{AC}^2} = \int_0^{\infty} \frac{4kTR d\omega}{2\pi [1 + \omega^2 R^2 C^2]} = \frac{kT}{C}$$

$$\text{or } \overline{Q_{AC}^2} (\text{Thermal}) = \overline{V_{AC}^2} C^2 = kTC$$



in Noise electrons

$$n = (\overline{N_{AC}^2})^{1/2} = \frac{\sqrt{kTC}}{e} = \underline{12.5 \sqrt{C(\text{fF})} e^-}$$

For example: $C = 25 \text{ fF}$

$$n = (\overline{N_{AC}^2})^{1/2} = 63 e^- \text{ r.m.s. noise electrons}$$

An alternative derivation:

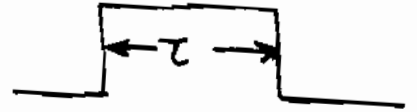
$$\frac{1}{2} kT = \frac{1}{2} C \overline{V_{AC}^2} \quad (\text{Equipartition Theorem})$$

$$\therefore \overline{V_{AC}^2} = kT/C$$

'Shot' Noise

Schottky Formula

$$\overline{I_n^2} = 2q I_0 \Delta f$$



If we allow electrons to accumulate over an integration time τ , then the r.m.s. charge accumulation becomes,

$$\overline{Q_n^2} = \int_0^\infty \left(\frac{\overline{I_n^2}}{\Delta f} \right) \tau^2 \left(\frac{\sin \pi f \tau}{\pi f \tau} \right)^2 df$$

$|F(\omega)|^2$ of filter function

$$\boxed{\overline{Q_n^2} = q I_0 \tau}$$

in Noise electrons

$$n = (\overline{N_n^2})^{1/2} = \sqrt{\frac{I_0 \tau}{q}} = 2.5 \sqrt{I_L (\mu A) \tau (ms)} e^-$$

example:

$$I_L = 2 \mu A \quad \tau = 10 \text{ ms}$$

$$n = 11 e^- \text{ r.m.s noise electrons}$$

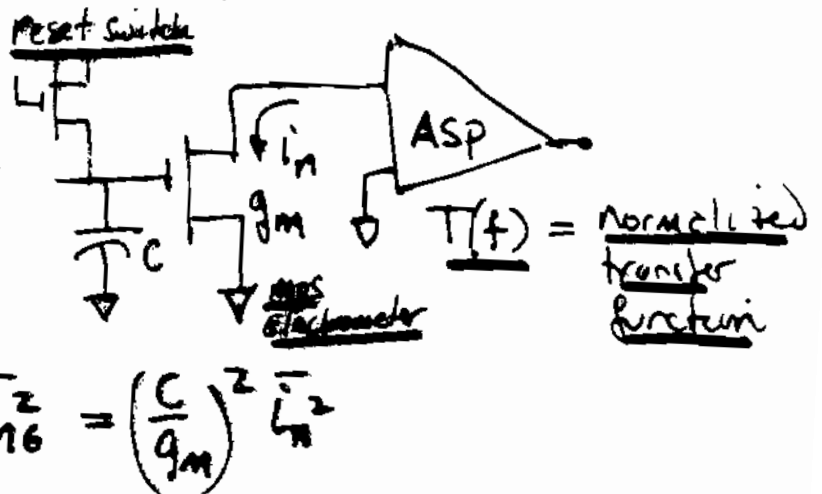
Photon Shot Noise

$$\boxed{\overline{Q_n^2} = q R_D E_{\text{eff}}}$$

Gate Referred Noise in a MOS Electrometer Amplifier

A MOS Transistor is often employed as the collection node for signal and noise charge in a sensor IC. The gate referred noise is calculated as follows:

The noise current i_n is referred to the input to create a noise charge



$$\overline{q_{n0}^2} = \left(\frac{C}{g_m}\right)^2 \overline{i_n^2}$$

If we use the transfer function $T(f)$ of the Analog Signal Processor (ASP), then the noise charge becomes,

$$\begin{aligned} \overline{q_{n0}^2} &= \left(\frac{C}{g_m}\right)^2 \int_0^\infty \left(\frac{\overline{i_n^2}}{\Delta f}\right) [T(f)]^2 df \\ &= \left(\frac{C}{g_m}\right)^2 B_{eff} \left(\frac{\overline{i_{n0}^2}}{\Delta f}\right) \end{aligned}$$

where

$$B_{eff} = \text{effective bandwidth} = \int_0^\infty \left(\frac{\overline{i_n^2}/\Delta f}{\overline{i_{n0}^2}/\Delta f}\right) [T(f)]^2 df$$

For example,

We consider the ASP
with the normalized
transfer function

$$T = T_1 T_2$$

$$= \frac{1 - e^{-sT_0}}{1 + s/\omega_A}$$

$$|T(\omega)|^2 = T(j\omega)T(-j\omega) = \frac{4 \sin^2(\omega T_0/2)}{1 + \omega^2/\omega_A^2}$$

Case (a)

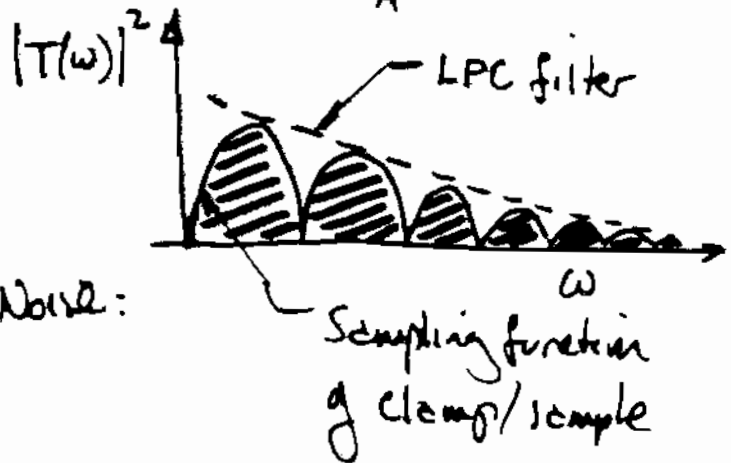
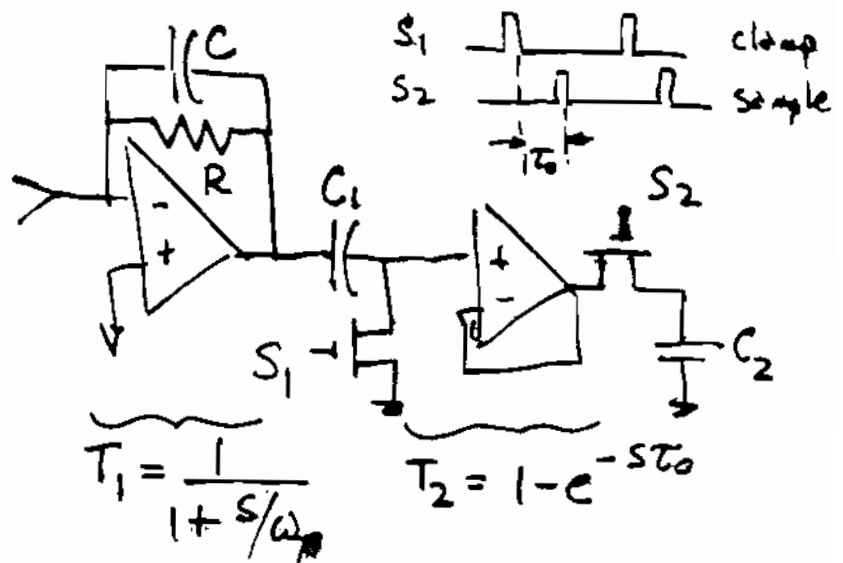
We consider a 'white'
Noise Spectrum of Shot Noise:

$$\therefore B_{eff} = \int_0^\infty |T(\omega)|^2 df$$

$$= \int_0^\infty \frac{4 \sin^2(\pi f T_0)}{1 + (f/\omega_A)^2} df = \pi f_A (1 - e^{-2\pi f_A T_0})$$

$$\text{and } \overline{q_n^2} = 2\pi g I_0 f_A (1 - e^{-2\pi f_A T_0}) \left(\frac{C}{g_m}\right)^2$$

$$= g I_0 \omega_A (1 - e^{-\omega_A T_0}) \left(\frac{C}{g_m}\right)^2$$



Case (b)

We consider a '1/f' noise spectrum caused by interface traps

$f_0 \sim$ Dit interface traps

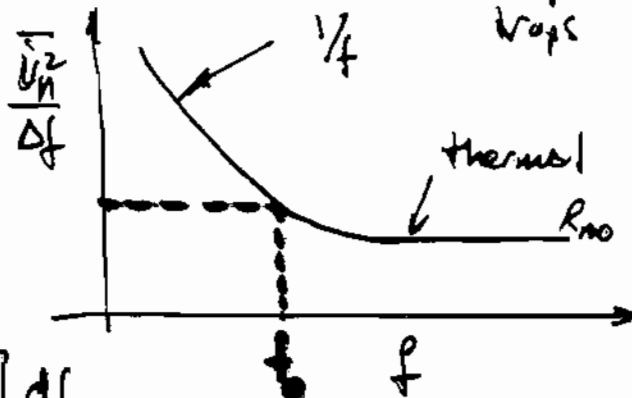
Effective noise resistance

$$R_n = R_{n0} \left(1 + \frac{f_0}{f}\right)$$

2nd bandwidth,

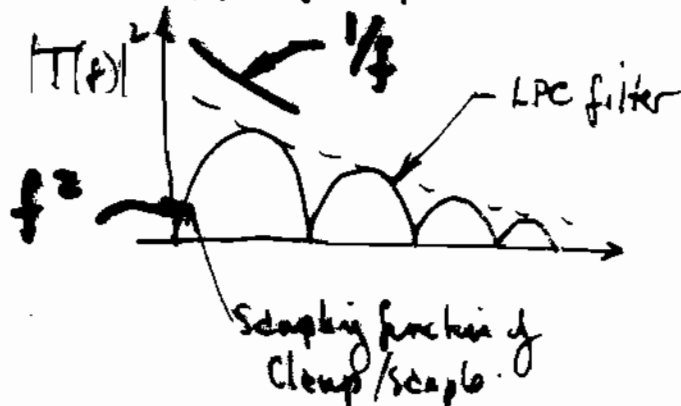
$$B_{eff} = \int_0^\infty \left(1 + \frac{f_0}{f}\right) \left[\frac{4 \sin^2 \pi f T_0}{1 + f^2 / f_A^2} \right] df$$

$$= \pi f_A \left(1 - e^{-2\pi f_0 T_0}\right) + (\pi f_A T_0)^2 f_0 \int_0^\infty \frac{\sin^2 x dx}{x [x^2 + (\pi f_A T_0)^2]}$$



double zero of clamp sample suppresses noise due to '1/f' spectrum

1/f noise is suppressed by clamp/sample operation



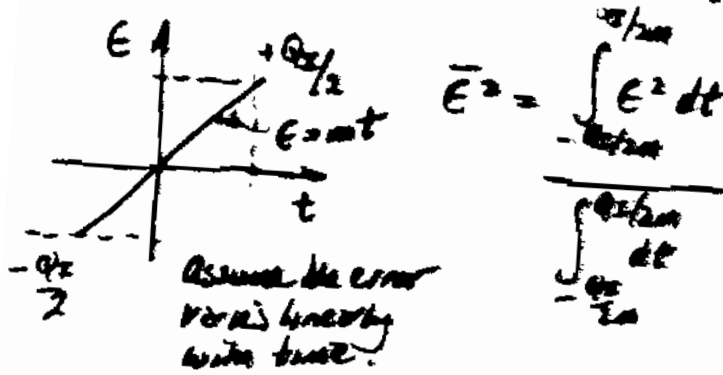
Total Noise Referred to Gate of MOS Transistor

$$\overline{Q_{nt}}^2 = \overline{Q_n}^2 (\text{Nyquist}) + \overline{Q_n}^2 (\text{shot}) + \overline{Q_n}^2 (\text{gen/rec + amplifier noise})$$

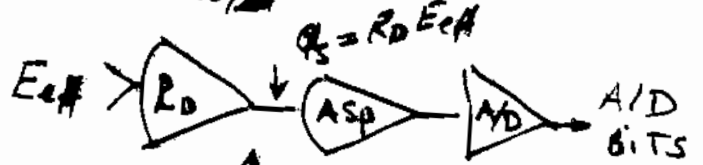
$$= \underbrace{kTC}_{\text{Nyquist}} + \underbrace{g I_0 T}_{\text{leakage shot}} + \underbrace{g R_D E_{eff}}_{\text{photon shot}} + \underbrace{\left(\frac{C}{g_m}\right)^2 B_{eff}}_{\text{gate referred amplifier noise + g-r noise}} \left(\frac{\overline{I_{no}}}{\Delta f}\right)$$

Total Noise Equivalent Signal (NES_{tot})

The Quantization by the A/D Converter in an Image Reversing System can influence the determination of the noise equivalent signal.



$$= \frac{m^2 t^3/3 \Big|_{-Q_E/2m}^{Q_E/2m}}{t \Big|_{-Q_E/2m}^{Q_E/2m}} = \frac{Q_E^2}{12}$$



Quantization Interval $\equiv \frac{\Delta E_{eff}}{\Delta(A/D) \text{ bits}} = Q_E$

$$\frac{S}{N} = \frac{Q_S}{[Q_N^2(m)]^{1/2}} = \frac{R_D E_{eff}}{\sqrt{\frac{(C \text{ in})^2}{g_m} + kTC + gI_c T + C^2 B_{eff} \left(\frac{H_{vis}}{\Delta t}\right) + g E_{eff} R_D + \frac{(Q_E R_D)^2}{12}}}$$

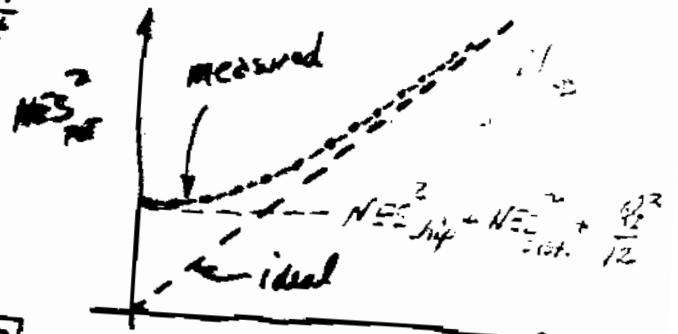
System input: $\frac{1}{f} \text{ Mas electron}$
 Photon shot noise: $\frac{(Q_E R_D)^2}{12}$
 No. of electrons: $E_{eff} (\mu J/m^2)$
 Transfer characteristic: $\Delta(A/D \text{ bits})$
 ΔE_{eff}

The noise equivalent signal is measured as (bits)_{mas} for a specified irradiance level (exposure density) and is given as

$$NES_{tot} (mas) = \frac{Q_E \text{ bits}}{(\mu J/m^2)} \Big|_{\text{bits per noise}} \left(\frac{\mu J/m^2}{\text{bit}} \right)$$

We can define this as the input exposure density at low light levels where the $\frac{S}{N} \ll 1$. Thus, neglecting system and A/D quantization noise, we have

$$NES_{lim} = \frac{1}{R_D} \sqrt{kTC + gI_c T + C^2 B_{eff} \left(\frac{H_{vis}}{\Delta t}\right)}$$



Slope at high exposure densities gives the detector responsivity, R_D .

Optimization of the Electrometer Amplifier

$$\frac{S}{N} = \frac{R_D E_{eff}}{\sqrt{kTC + \gamma I_0 \tau + \left(\frac{C}{g_m}\right)^2 B_{eff} \left(\frac{i_{no}^2}{\Delta f}\right)}}$$

where we consider the optimization with respect to NES_0 .

mos amplifier

$$C = C_A + C_p \leftarrow \text{parasitic (interconnect + junction)}$$

$$\frac{\bar{I}_{no}^2}{g_m^2 \Delta f} = \frac{\bar{V}_{no}^2}{\Delta f} = 4kTR_D \left(R_D \approx \frac{\alpha}{4C_A} \right) \text{ From the theory of op-amp limits on } \frac{1}{f} \text{ noise.}$$

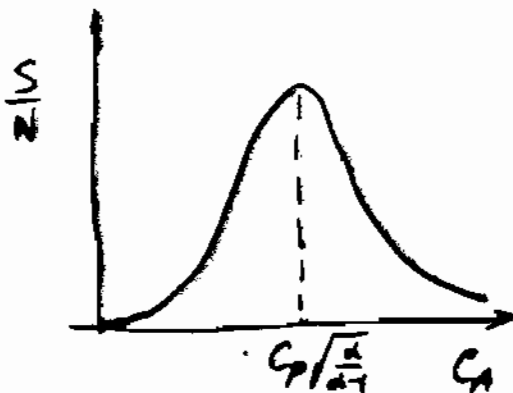
thus,

$$\frac{S}{N} = \frac{R_D E_{eff}}{\sqrt{kT(C_A + C_p) + \gamma I_0 \tau + \frac{(C_A + C_p)^2 kT \alpha}{C_A}}}$$

We now maximize the $\left(\frac{S}{N}\right)$ ratio with respect to C_A ,

$$\frac{\partial}{\partial C_A} \left(\frac{S}{N}\right) = 0 \text{ gives } kT - kT \alpha \left[\frac{C_A^2 - C_p^2}{C_A^2} \right] = 0$$

$$\Rightarrow C_A = C_p \sqrt{\frac{\alpha}{\alpha - 1}} \rightarrow C_p \text{ for low Nyquist noise contributions}$$

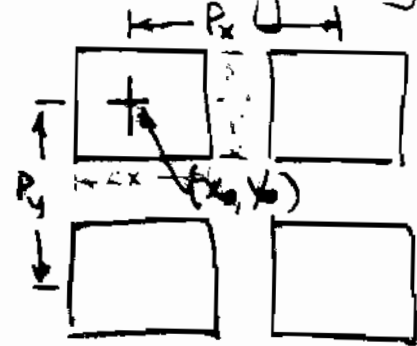


Thus, for a given parasitic capacitance, C_p , we should try to design the amplifier, such that $C_A = C_p$.

Bandwidth Considerations

We consider the design of a solid-state imaging array which employs discrete sensors.

We use the concept of a modulation transfer function (M.T.F.) which is the pixel response to a sinusoidal spatial frequency input.



$$H(x) = \text{irradiance of the input sinusoid} = H_0 (1 + A \cos 2\pi f_s x)$$

where f_s is the spatial frequency of the input signal (1/pixel). The maximum spatial frequency which may be processed by the sensor array is $f_s(\text{max}) = \frac{1}{2p}$, which is set by the Nyquist Sampling theorem. The uncertainty in the measurement may be written as,

$$\Delta H(x_0) = H(x_0) - H_m(x_0) = H(x_0) - \frac{1}{\Delta x} \int_{x_0 - \frac{\Delta x}{2}}^{x_0 + \frac{\Delta x}{2}} dx H(x)$$

where

$$H_m(x_0) = H_0 (1 + M \cos 2\pi f_s x_0)$$

$$M \triangleq A \frac{\sin 2\pi f_s \Delta x}{\pi f_s \Delta x}$$

with a definition of the Modulation Transfer Function (M.T.F.)

$$\text{M.T.F.} \triangleq \frac{M(f_s)}{M(0)} = \frac{\sin \pi f_s \Delta x}{\pi f_s \Delta x} = \frac{\sin \left(\frac{\pi \Delta x f_s}{2p f_s(\text{max})} \right)}{\frac{\pi \Delta x f_s}{2p f_s(\text{max})}} \quad \begin{matrix} \text{Geometric} \\ \text{M.T.F.} \end{matrix}$$

We treat the design of a line array with

$$M.T.F._x = \frac{\sin \pi f_s \Delta x}{\pi f_s \Delta x}$$

$$M.T.F._y = \frac{\sin \pi f_s \Delta y}{\pi f_s \Delta y}$$

$$\cdot \underbrace{\frac{\sin \pi f_s \Delta \tau}{\pi f_s \Delta \tau}}_{\text{Sincar function}}$$

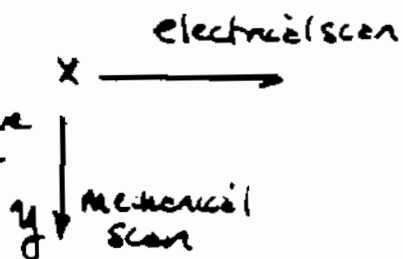


Figure-of-Merit (F.M.)

We formulate a figure-of-merit for a solid state imager as follows:

$$F.M. = \frac{S}{N} \times M.T.F.$$

where we will consider the intrinsic F.M. associated with the chip and the geometrical M.T.F.

Case (1) Noise is determined by the pre-amplifier and is independent of the sensor area.

$$\therefore \frac{S}{N} \sim \Delta x \Delta y$$

The effective resolution (M.T.F.) is taken as the geometric mean of its components and we write

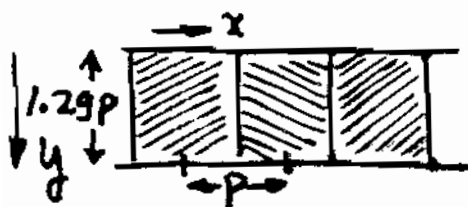
$$F.M. = \Delta x \Delta y [M.T.F._x M.T.F._y]^{\frac{1}{2}}$$

We maximize the F.M. at the Nyquist Limit $f_s = f_s(\max) = \frac{1}{2p}$ and let $\Delta \tau = p$ to conserve bandwidth,

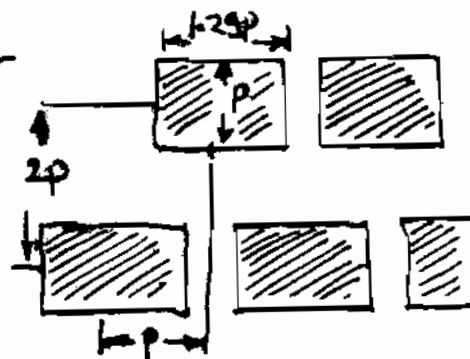
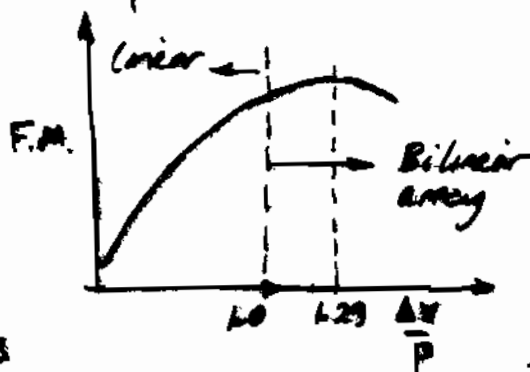
$$F.M. \Big|_{f_s(\max)} = \Delta x \Delta y \left[\frac{\sin\left(\frac{\pi \Delta x}{2p}\right)}{\pi \frac{\Delta x}{2p}} \right]^{\frac{1}{2}} \left[\frac{2}{\pi} \frac{\sin\left(\frac{\pi \Delta y}{2p}\right)}{\frac{\pi \Delta y}{2p}} \right]^{\frac{1}{2}}$$

Differentiation with respect to Δy yields,

$$\tan \frac{\pi \Delta y}{2p} = -\frac{\pi \Delta y}{2p} \Rightarrow \Delta y = 1.29p$$



a true linear array with contiguous pixels and a 2P off-set.



a Bilinear Array for maximum performance with a 2P off-set.

Cartography

In applications, such as cartography, equal resolution is required in both x and y directions.

We set $MTF_x = MTF_y$ at $f_s = f_s(\max)$ and let $\Delta x = p$ to conserve bandwidth.

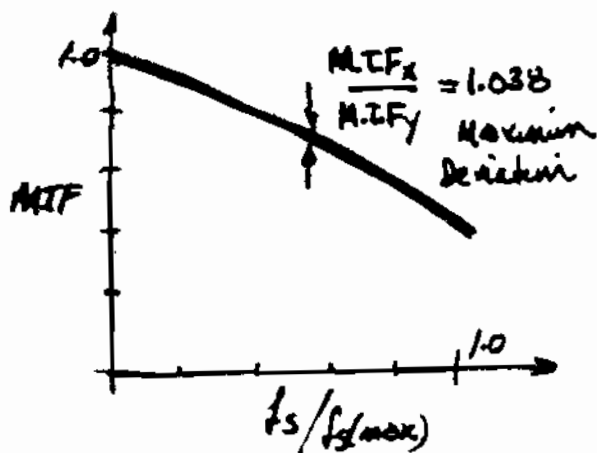
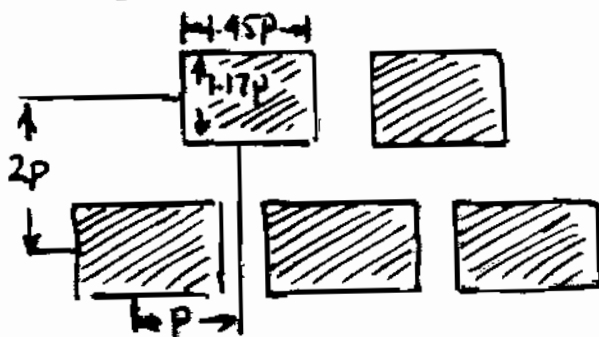
$$\frac{\text{Sinc}\left(\frac{\pi \Delta x}{2p}\right)}{\frac{\pi \Delta x}{p}} = \frac{2}{\pi} \frac{\text{Sinc}\left(\frac{\pi \Delta y}{2p}\right)}{\frac{\pi \Delta y}{2p}}$$

Constraint

$$F.M. = \frac{4 \Delta x p}{\pi^2} \text{Sinc}\left(\frac{\pi \Delta y}{2p}\right)$$

We maximize the F.M. subject to the constraint and find

$\Delta x = 1.45p$	pre-amplifier
$\Delta y = 1.17p$	NVA limited



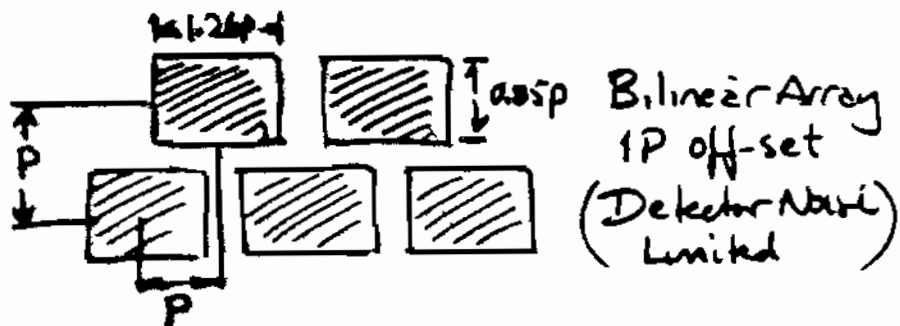
Case(2) If the preamplifier noise is very low, then the noise can be limited by the sensor.

$$\therefore \frac{S}{N} \sim (\Delta x \Delta y)^{1/2}$$

and we find the equal resolution array design becomes,

$$\Delta x = 1.26p \quad \text{Detector Noise Limited}$$

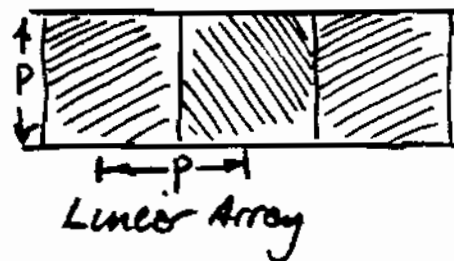
$$\Delta y = 0.85p \quad \text{Detector Noise Limited}$$



For the linear array we cannot have equal resolution but we can ensure bandwidth to find,

$$\Delta x = \Delta y = p$$

Detector Noise Limited



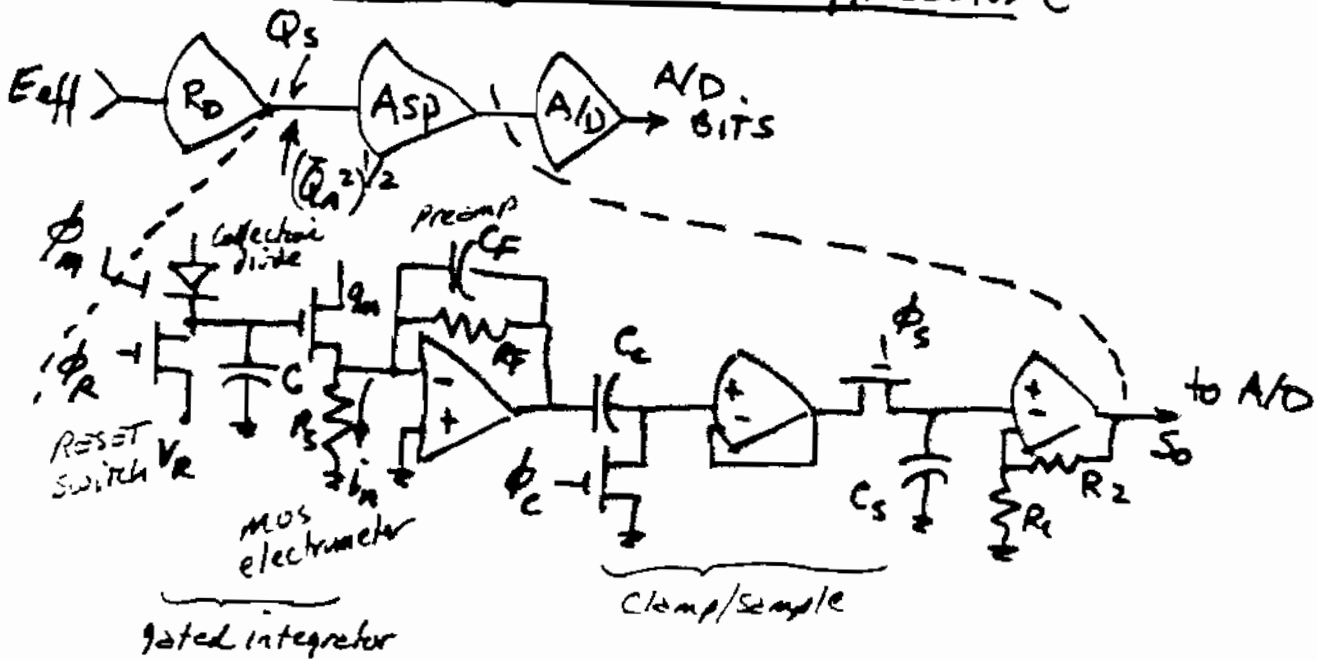
Summary of Design for Line Arrays

Type	Preamplifier Limited	Detector Limited
Maximum Performance Linear Array	$\Delta x = p$ $\Delta y = 1.29p$ 0p off-set	$\Delta x = \Delta y = p$ 0p off-set
Equal resolution Bilinear Array	$\Delta x = 1.45p$ $\Delta y = 1.17p$ 2p off-set	$\Delta x = 1.26p$ $\Delta y = 0.85p$ 1p off-set

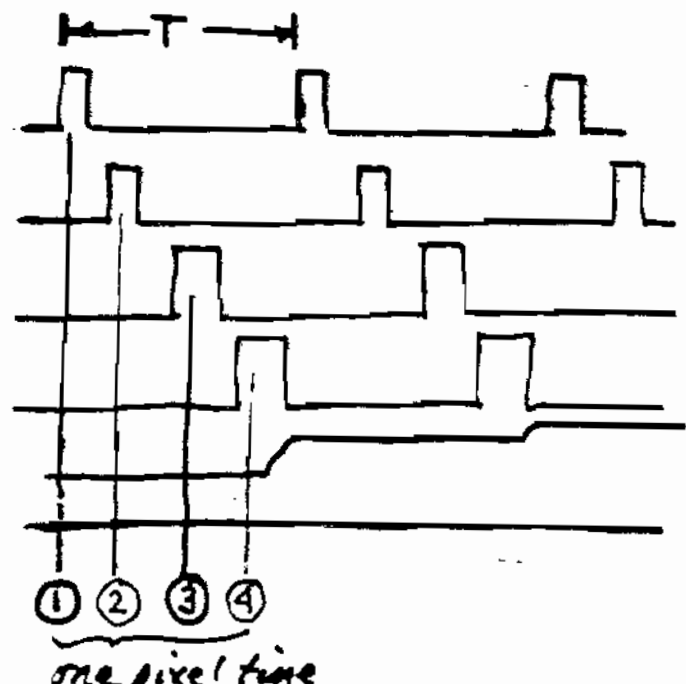
NOISE SUPPRESSION TECHNIQUES

- **Integration (storage)**
- **Differential amplifier (common mode rejection)**
- Correlation (e.g. matched filter)
- **Synchronous detection (AM demodulation, PLL)**
- Adaptive filtering (noise canceller)
- **Correlated double sampling (CDS)**
- **Ring Junction Gate Detection (RJG)**

Correlated Double Sampling (CDS) Method for Noise Suppression



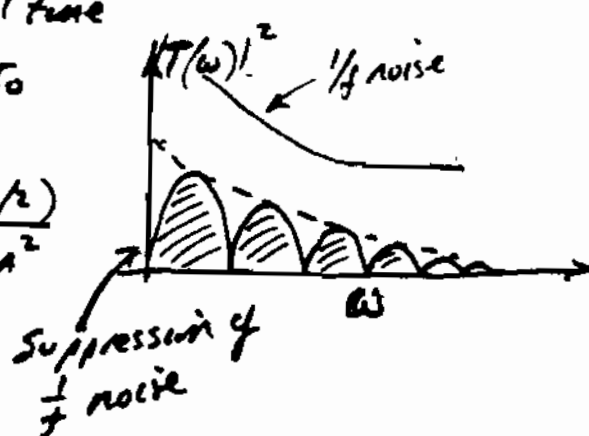
- ① Reset C with ϕ_R
- ② Clamp Reset and Noise with ϕ_C
- ③ Mux signal onto Capacitor C with ϕ_M
- ④ Sample signal (less reset noise) with ϕ_S



$$T_{preamp} = \frac{1}{1 + s/\omega_n}$$

$$T_{cls} = 1 - e^{-sT_0}$$

$$|T(\omega)|^2 = T_{preamp} T_{cls} = \frac{4s\omega_n^2 (\omega T_0/2)}{1 + \omega^2/\omega_n^2}$$



- Removes kTC noise
- Suppresses 1/2 noise

CONCLUSIONS

- Design of solid-state imagers may be achieved by maximizing the figure-of-merit = $(S/N) \cdot (MTF)$ with respect to the pixel dimensions
- Reset and $1/f$ noise cancellation may be accomplished with a correlated double sampling technique
- The (S/N) may be maximized by designing the output amplifier capacitance, C_a , to match the parasitic capacitance, C_p