DESIGN OF MINIMUM-PHASE CHARGE-TRANSFER TRANSVERSAL FILTERS

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Abstract :

The interest of minimum-phase charge-transfer transversal filters is pointed out in terms of sensitivity. When used in communication networks, such filters are indispensable for meeting the CCITT group-delay requirements. An experimental example for time division multiplex equipment is described.

Charge-transfer devices proved to be very efficient for designing non recursive discrete-time analogue filters. Extensive work has been carried out on the synthesis of linear-phase (l.p.) transversal filters[1-3]. For that case, minimax solutions are well characterised and several approaches are possible. However, when applied to communications, such filters admit a group delay that is rarely consistent with CCITT requirements. This latter condition implies that the linear-phase specification be removed and replaced by minimum phase (m.p.) operation. Owing to the development of sophisticated charge-transfer transversal filters, this problem is no longer academic[4] and its solution is required, for example, in the synthesis of filters for time- or frequency-division multiplex.

The synthesis of minimum-phase transversal filters has been theoretically solved by Herrmann and Schuessler[5] using linear-phase synthesis as a first step. This method necessitates finding the roots of a polynomial whose degree is equal to the order of the filter. As pointed out by Schmidt and Rabiner[6], this is a difficult approach in numerical analysis for sophisticated filters, but a direct synthesis is also possible[7].

The frequency response of any transversal filter is computed from the equation

$$s(z) = \sum_{k=0}^{N-1} A_k z^{-k} \qquad (1)$$

where $z = \exp(j\omega\tau)$, ω being the angular frequency and τ the clock period. The synthesis problem is then to find the N parameters $A_k (k=0,1,\ldots,N-1)$. The l.p. synthesis is obtained by forcing

$$A_k = A_{N-k-1} \qquad (2)$$

Such a symmetrical weighting leads to a real amplitude response, followed by a pure delay of half the duration of the impulse function, i.e. a linear-phase response. In that case, the parameters A_k are synthesised by several techniques including windowing of an infinite impulse response[8] and digital algorithms such as the Remez exchange algorithm[3] or linear programming[2].

Of course, a more efficient synthesis can be obtained by removing the constraints of eqn.2. The advantage is clear in terms of actual group delay, since minimum phase implies minimum delay. In addition, Rabiner et al[9] have shown that, when using m.p. filters, the gain in terms of number of taps depends upon the relative bandwidth of the filters : the larger the bandwidth, the larger the gain in number of taps.

Without loss of generality, we will first assume that $\max |s(z)| = 1$ on the unit circle.

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Clearly, the amplitude response is defined by :

$$|s(z)|^2 = s(z)\, s(z^{-1}) = S(\zeta) \quad (3)$$

where

$$\zeta = (z + z^{-1})/2$$

Poles and zeros of insertion loss are easily translated into zeros of $S(\zeta)$ and zeros of $1-S(\zeta)$ respectively with $\zeta \in [-1, +1]$. We may then factor $S(\zeta)$ and $1-S(\zeta)$ as

$$S(\zeta) = F(\zeta)\, G(\zeta) \quad (4)$$

$$1-S(\zeta) = H(\zeta)\, Q(\zeta) \quad (5)$$

where $F(\zeta)$ is a polynomial which accounts for all the zeros of $S(\zeta)$ on $[-1, +1]$ and $H(\zeta)$ is a polynomial which accounts for all the zeros of $1-S(\zeta)$ on $[-1, +1]$.

The polynomials $F(\zeta)$ and $H(\zeta)$ are completely defined by their zeros ζ_k and their corresponding multiplicities ν_k. Clearly if $\zeta_k \in]-1, +1[$ the multiplicity is even, since S and 1-S are strictly positive. If $\zeta_o = 1$ or $\zeta_\infty = -1$, the multiplicity ν_o or ν_∞ is arbitrary. Finally, we have to find the unknown polynomials G and Q defined by

$$FG + HQ = 1 \quad (6)$$

This Bezout identity implies that F and H be mutually prime, which clearly holds. It is easily proved that if (G_o, Q_o) is one solution, all other solutions are

$$G = G_o + \lambda H$$
$$Q = Q_o - \lambda F \quad (7)$$

where λ is an arbitrary polynomial. Let N_1 and N_2 be the degrees of H an F respectively. From Eqs 6-7 it is seen that G can be chosen with a degree less or equal to $N_1 - 1$ and Q with a degree less or equal to $N_2 - 1$. In order than G and Q be relevant solutions, it is necessary that they have no zero on $[-1, +1]$. Unfortunately, this condition does not hold automatically and this has to be checked. More generally, we may choose λ of degree zero or more in order to eliminate the zeros on $[-1, +1]$ of G or Q. (Again, this is not always possible for any choice of the zeros ζ_k).

Finally, when G has been found with its lowest degree the filter of eqn.4 is solution of the problem.

To recover the z - transform, we must split F and G as

$$F = f(z)\; f(z^{-1})$$

$$G = g(z)\; g(z^{-1})$$

Clearly, a factor $(\zeta \pm 1)$ in F gives rise to a factor $(z^{-1} \pm 1)$ in f and a factor $(\zeta - \zeta_k)^2$ gives rise to a factor $(z^{-2} - 2\zeta_k z^{-1} + 1)$: the expansion of f(z) is completely known.

The derivation of g (z) requires the extraction of the complex roots of the $G(\zeta)$ polynomial which is generally of degree $N_1 - 1$.

Each pair of complex roots ζ_ℓ and ζ_ℓ^* of G is translated into a pair of roots z_ℓ and z_ℓ^* located inside the unit circle for minimum phase. Each real root z_ℓ of G (with $|\zeta_\ell| > 1$) is similarly translated into a real root z_ℓ of g (z) (with $|\zeta_\ell| < 1$).

Finally, we have : $s(z) = f(z)\, g(z)$

On the other hand, we can select a linear phase filter from $S(\zeta)$ only if $G(\zeta)$ is the square of a polynomial. In that case the number of taps is the same in both implementations and with the same amplitude variations, and the advantages of m.p. synthesis can be expressed in terms of sensitivity. For any transversal filter, we will assume that the N weighting coefficients are random independent variables with the same standard deviation σ_A.

Then clearly the signal variance is

$$\sigma_S^2 = \overline{\{s(z)s(z^*) - |\bar{s}(z)|^2\}} = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} \overline{(A_k A_l - \bar{A}_k \bar{A}_l)}\, z^{k-l}$$

and finally, $\sigma_S = N^{1/2}\sigma_A$

(the line stands for mathematical expectation). Owing to this sensitivity, the maximum insertion loss in the stopband can be evaluated for a low pass filter as

$$IL_{max} \approx -20 \log_{10} \frac{\sigma_S}{|S_{max}|} \approx -20 \log_{10} \left(\frac{N^{1/2}\sigma_r A_{max}}{\sum A_k} \right)$$

where $\sigma_r = \sigma_A / A_{max}$ is the relative standard deviation, A_{max} is the maximum modulus of the coefficients A_k and $|S_{max}|$ is the maximum value of $|s(z)|$ reached for $z \approx 1$ $(\omega \approx o)$.

Direct comparison between optimum l.p. and m.p. filters is not very easy since ther is generally no connection between the two syntheses, and the question arises of what is to be compared. When the response is obtained by cascading two identical filters, the overall transmittance admits only double roots in the z-plane and can then be implemented in both l.p. and m.p. devices with the same number of taps. Neither synthesis is optimum, but a direct comparison in terms of sensitivity is then straighforward. This comparison procedure has been chosen starting from an 88 taps m.p. prototype as follows :

a) Synthesise an m.p. prototype with N taps : $s_0(z)$

b) Compute $s_0(z)\, s_0(1/z)$ and implement a l.p. filters $s_{\ell p}(\omega)$ (cf. Fig.1a).

c) Compute $[s_0(z)]^2$ and implement an m.p. filters $s_{mp}(\omega)$ (cf. Fig. 1b).

d) Use a Monte Carlo variation on the weighting coefficients on both $s_{\ell p}$ and s_{mp} and compute the sensitivity.

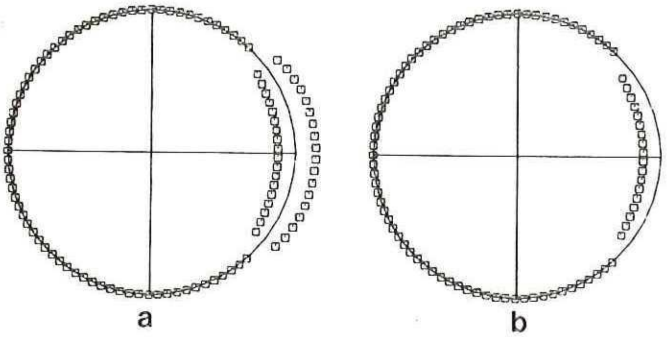


Fig. 1. Zeroes - pattern of a l.p. filter (a) and a m.p. filter (b).

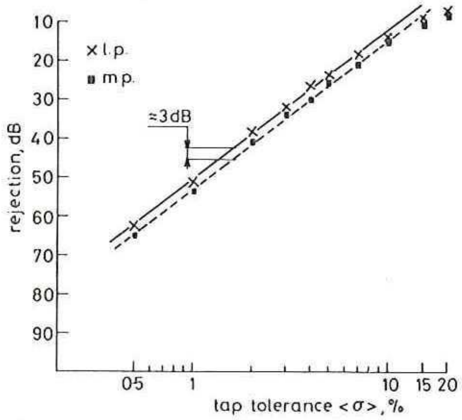


Fig. 2. Monte-Carlo simulation of l.p. and m.p. filters.

From a theoretical point of view, if the prototype coefficients are R_k $(k = 0, 1, 2, \ldots, n-1)$ it is easily seen that the coefficients $A_k^{\ell p}$ of the linear phase filter are :

$$A_k^{lp} = \sum_{i-l=k} R_i R_l \le \left(\sum_{i=0}^{n-1} R_i^2\right)^{1/2} \left(\sum_{l=0}^{n-1} R_l^2\right)^{1/2} \qquad (k = -n, \ldots + n)$$

(we have used Schwarz's inequality). Then $A_0^{lp} = A_{max}^{lp} = \sum_{l=0}^{n-1} R_l^2$

Similarly, we have for the m.p. filter $A_k^{mp} = \sum_{i+l=k} R_i R_l \le \sum_{i=0}^{k} R_i^2 \quad (k = 0, 1, \ldots, 2n)$

(using again Schwarz's inequality) and finally

$$A^{lp}_{max} \geq A^{mp}_{max}$$

Since we have (for the low pass filter)

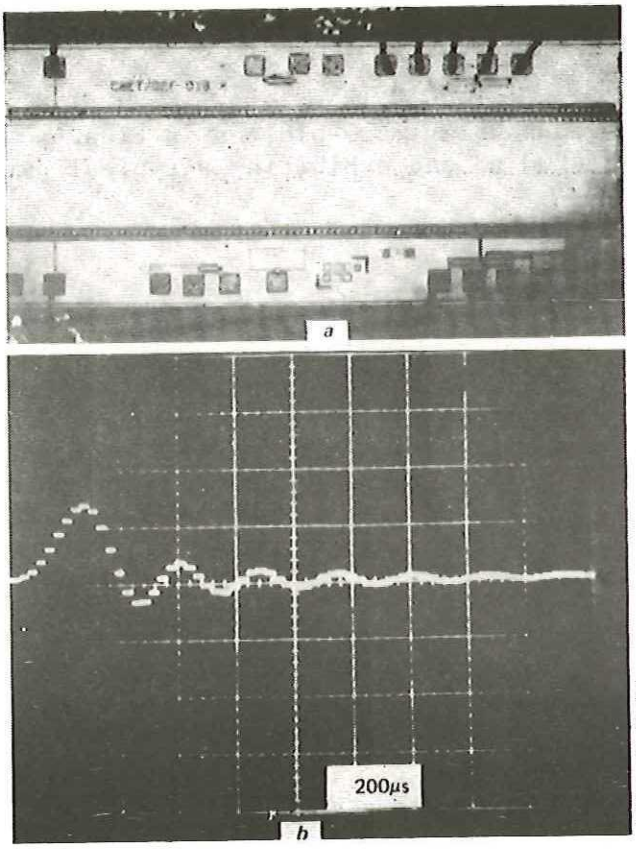
$$S_{max} \approx S^{lp}(1) = S^{mp}(1) = \sum_{k=-n}^{+n} A^{lp}_k = \sum_{k=0}^{2n} A^{mp}_k$$

the above inequality implies that there is always a better sensitivity for the m.p. filter than for the l.p. filter. The improvement can be evaluated as

$$\Delta IL = 20 \log_{10}\left(\frac{A^{mp}_{max} \sum A^{lp}_k}{A^{lp}_{max} \sum A^{mp}_k}\right)$$

In the above filters with (2 x 88 +1) taps we have ΔIL = 2.37 dB. To check the sensitivity of both filters a Monte Carlo simulation hasbeen used. The results are summarised in Fig.2. In the passband, the sensitivities are very similar and rather low for both syntheses. On the other hand, in the stopband the m.p. synthesis shows an improvement in the rejection level of roughly 3 dB for the same inaccuracy of the taps, making clear that the use of an m.p. filter leads to a lower sensitivity than an l.p. synthesis.

Such a minimum-phase charge-transfer transversal filter has been implemented, meeting the requirements of the CCITT pulse-code-modulation voice-channel filter. To be able to compare minimum and linear-phase filters we have chosen, at a clock frequency of 32 KHz, the same number of taps (88) for the two designs. Otherwise, only about 60 taps would be needed to fulfil the conditions with the minimum-phase design.



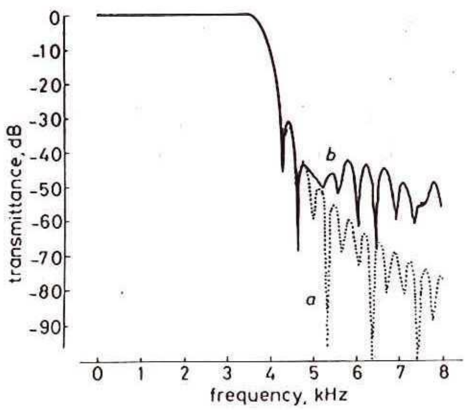


Fig. 4. M.P. filter frequency response
a) computed transmittance
b) experimental transmittance (10 dB/div.).

Fig. 3. Impulse response of the m.p. filter
a) photograph ot the c.c.d.m.p. filter
b) experimental impulse response.

We used the conventional n-channel double-polysilicon-gate technology on a p <100> substrate of $R_0 = 20\,\Omega$.

Fig.3.a shows a phtograph of the device, where the theoretical impulse response is represented by the profile of the splits in the electrodes. The experimental impulse response is displayed in Fig.3b, corresponding very fairly to the same shape and showing clearly the asymetrical response with low group delay.

The calculated and observed filter frequency responses are plotted in Figs 4a and b and demonstrate that a very good agreement can be obtained except for beyond 50 dB rejection, a limit probably caused by random technological errors (c.f. Fig.2 : tap tolerance about 1 %). An enlarged part of the insertion loss against frequency is shown in Fig.5a (compared with theory in dotted line), showing ripples less than ±0.1 dB in the passband of the filter. With the same scales, Fig. 5b shows the observed transmittances of 5 filters obtained without modification of the settings and demonstrating a good reproducibility, well inside the CCITT requirements (±0.25 dB).

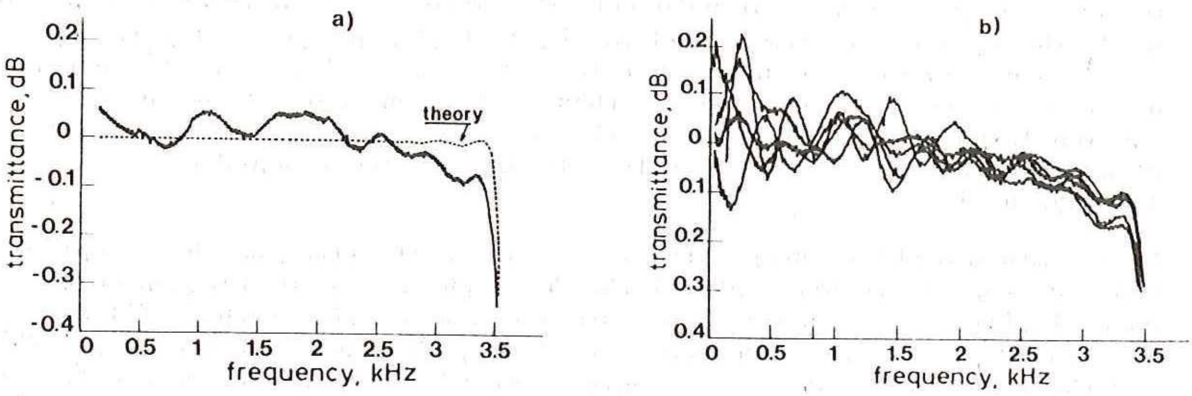


Fig. 5. Experimental transmittance (0.1 dB/div)
a) best filter compared with theory
b) 5 filters without resetting.

Finally, the computed and observed group delays are plotted in Figs.6a and b starting from the low value of 200 nS and exhibiting a typical parabolic shape.

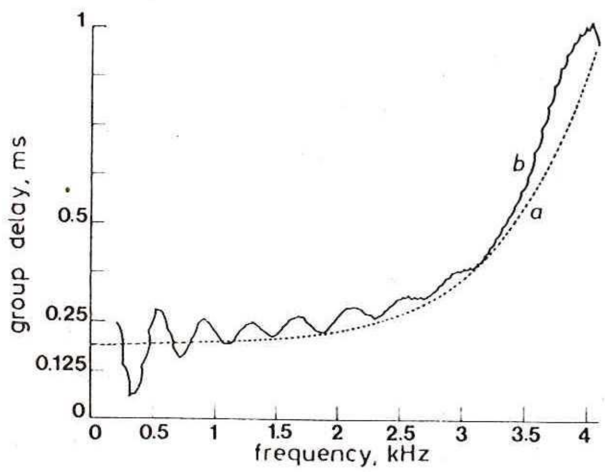


Fig. 6. M.P. filter frequency response
a) computed group delay
b) observed group delay.

In conclusion, we have outlined the interest of minimum-phase-design of c.c.d. filters in terms of both number of taps and sensitivity. In addition, such synthesis has to be used for more sophisticated communication filters, for example frequency-division-multiplex filters.

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