A TIME DOMAIN ANALYSIS OF VIDEO INTEGRATORS

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ABSTRACT. It has been shown that video integrators can be realised with charge coupled devices (CCD's). Such a realisation exploits the advantages of a CCD in that it is a sampled data device and combines analogue operation with the flexibility of digital techniques including the precise control of delay time. A video integrator serves to improve the signal-to-noise ratio of repetitive signals by integrating the signals in a recirculating delay line.

Unfortunately smearing or charge transfer inefficiency places a serious limitation on the performance of such integrators. The effect of smearing is to generate and feed the growth of secondary signals in the elements of the CCD following that containing the primary or wanted signal.

This paper presents a detailed analysis of the growth of secondary signals in CCD delay line integrators. Calculated values of the relative magnitudes of the primary and first two secondary signals are given as functions of the loop gain, transfer efficiency and number of transfers. On the basis of this analysis several techniques are proposed for reducing secondary growth whilst maintaining the required loop gain and hence the desired signal-to-noise ratio improvement.

1. THE VIDEO INTEGRATOR

If a simple recursive filter is operated at a frequency at which signal build up occurs, it can be considered as an integrator. Of particular importance is the processing of repetitive pulsed signals, as may be obtained in a radar system. If the time between pulses is equal to the delay time, τ, the recirculated and incident pulses will add. The similar growth of noise is slower because it adds in a root mean square manner so there is enhancement of the signal to noise ratio. The advantage of a CTD system is that the precise control of the delay time, inherent in the present digital techniques, is retained, but there is a simpler compatibility between pulsed signals and the sampled data, analogue operation of the CTD[1]. It can be shown that charge transfer inefficiency modifies the transfer function of the recursive filter but in this system the steady state frequency domain representation of the filter is no longer useful, rather it is the growth of signals in time and the understanding of charge transfer inefficiency effects in the time domain that is of interest.

The effect of charge transfer inefficiency is to generate and feed the growth of secondary signals[2] in the elements of the CTD following that containing the primary or wanted signal so that an error is added to any signal contained in the next time cell. Once the secondary signal grows above the noise level in that element of the device, it effectively destroys some of the signal to noise ratio enhancement obtained through integration. The secondaries can be reduced by decreasing the loop gain but this in turn also reduces the possible signal to noise ratio improvement.

Consider the build up of signals in the time domain when the network of Fig. 1 is fed with a train of equal height pulses with a pulse repetition frequency equal to the inverse of the delay time. The delay line has an impulse response given by

$$A + BZ + CZ^2 + \ldots \quad (1)$$

where Z is a delay operator, $e^{-j\omega T_1}$ representing one stage of delay, T_1 in the CTD delay line (one stage contains p elements in a p phase CTD) and B, C etc. are amplitudes of the spurious signals.

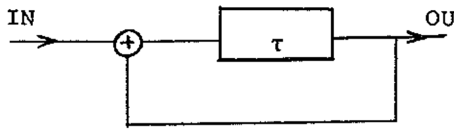

FIG. 1

A fraction, K, of the output from the first input pulse is then fed back to the input so that the input signal becomes

$$(1 + KA) + KBZ + KCZ^2 \ldots$$

After m such circulations the primary signal builds up as

$$S_A = A(1 + KA + (KA)^2 + (KA)^3 + \ldots (KA)^{m-1} \quad (2)$$

the secondary and tertiary as

$$S_B = B(1 + 2KA + 3(KA)^2 + \ldots\ n(KA)^{m-1} \quad (3)$$

$$S_C = C(1 + 2KA + 3(KA)^2 + \ldots\ n(KA)^{m-1}) + KB^2(0+1 + 3KA + 6(KA)^2 + \ldots \frac{m(m-1)}{2}(KA)^{m-2}) \quad (4)$$

For positive values of K any one spurious output builds up faster than the ones leading it, although the magnitude is controlled by the factors A, B, C, because it feeds on all the leading outputs. In the case of the primary and secondary for example, the primary builds up at a rate which is dependent on the fact that the increasing pulses are of unit amplitude, whilst the secondary build up depends on the amplitude of the primary which is continually growing to a value which is greater than unity.

The magnitudes of the signals after an infinite number of circulations can be written

$$S_A = \frac{A}{1-KA} \qquad (5)$$

$$S_B = \frac{B}{(1-KA)^2} \qquad (6)$$

$$S_C = \frac{C}{(1-KA)^2} + \frac{KB^2}{(1-KA)^3} \qquad (7)$$

Equations 5 and 6 are similar in form to those derived by Urkowitz(3) for secondary responses in conventional analogue delay line integrators.

An infinite number of circulations will be considered as these give the 'worst case' results.

2. CTD IMPULSE RESPONSE

In order to compare the relative magnitudes of the output signals it is necessary to determine the coefficients A, B, C. For a CTD delay line the impulse response can be written as (4)

$$\delta^n + Z(n\varepsilon\delta^n) + Z^2\left(\frac{n(n+1)}{2}\varepsilon^2\delta^n\right) \ldots\ldots$$

$$Z^j \, C_j^{j+n-1} \, \varepsilon^j \delta^n \qquad (8)$$

where $\delta = 1-\varepsilon-\ell$, δ is the charge transferred at each stage, ε is the charge residual, ℓ is the charge loss per transfer, C_j^{j+n-1} is the binomial coefficient, and n is the number of transfers (elements or stages in a single phase device). The first three terms correspond to the coefficients, A, B, C.

In order to simplify this equation assume that the device is operated at a clock frequency where the loss of charge can be neglected (i.e. $\delta = 1-\varepsilon$, this then is strictly only true for a CCD).

To a good approximation

$$\delta^n = 1 - n\varepsilon \qquad (9)$$

provided that $n\varepsilon$ is small (<0.1).

Assuming that $(n)(n+1) = n^2$, equations 5, 6, 7 may now be written as

$$S_A = \frac{1-n\varepsilon}{1 - K(1-n\varepsilon)} \qquad (10)$$

$$S_B = \left[\frac{n\varepsilon}{1 - K(1-n\varepsilon)}\right] \cdot S_A \qquad (11)$$

$$S_C = \left[\frac{(n\varepsilon)^2}{2(1 - K(1-n\varepsilon))} + \frac{K(n\varepsilon)^2(1-n\varepsilon)}{\left[1 - K(1-n\varepsilon)\right]^2}\right] \cdot S_A \qquad (12)$$

The magnitudes of the output signals, $S_{A,B,C}$ are shown in Fig. 2 as functions of $n\varepsilon$ for given values of K. If the device was perfect ($n\varepsilon$=0) then S_B, S_C would be zero and S_A reduces to 1/1-K). Consider the curves for K = 0.99 in Fig. 2. For low values of $n\varepsilon$, S_A approaches the ideal value and the amplitudes of the secondaries are low. As $n\varepsilon$ increases two effects can be observed. Firstly the secondaries build up to magnitudes close to that of the primary. The secondary-primary ratios approach unity. Secondly the magnitude of the primary becomes increasingly lower than the ideal value. With reference to equation 10 this can be attributed to an effective reduction in the loop gain. The gain has fallen from K to an effective value $K' = K(1-n\varepsilon)$. These

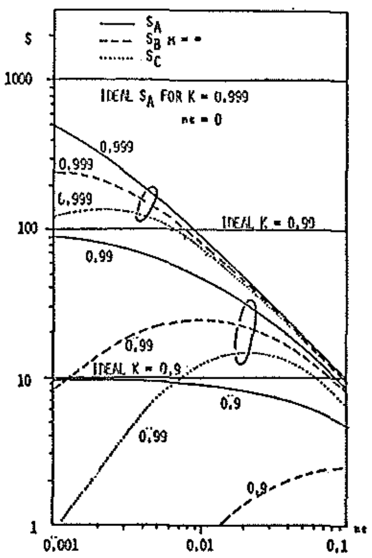

FIG. 2

The magnitudes of the primary S_A and the first two secondary signals S_B, S_C as a function of the loop gain K and the overall transfer efficiency for an infinite number of circulations.

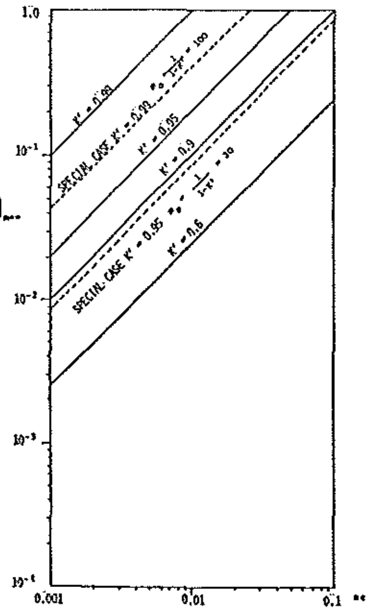

FIG. 3 (a)

The broken lines indicate the results for a finite number of circulations which give an integration time equal to the time constant of the integrator.

effects are exaggerated as K approaches unity. For increasing K an even smaller n is required to maintain the same level of $(\frac{S_{B,C}}{S_A})$.

3. DESIGN CONSIDERATIONS

In practice to achieve a desired signal to noise ratio improvement, S_A has to grow to the required magnitude. The loop gain must be maintained. Equation 10 can then be rewritten in terms of K'. This is perhaps a more useful form of the equations for design purposes as it is K' that sets the performance of the integrator. The ratios $S_{B,C}/S_A$ are shown plotted in Fig. 3 as functions of the effective loop gain. Note that the ratios can now exceed unity because for a given K' and nε, K may be greater than unity.

The results for $\frac{1}{1-K'}$ circulations (the filter time constant) are shown in Fig. 3 for comparison with the case of $m = \infty$.

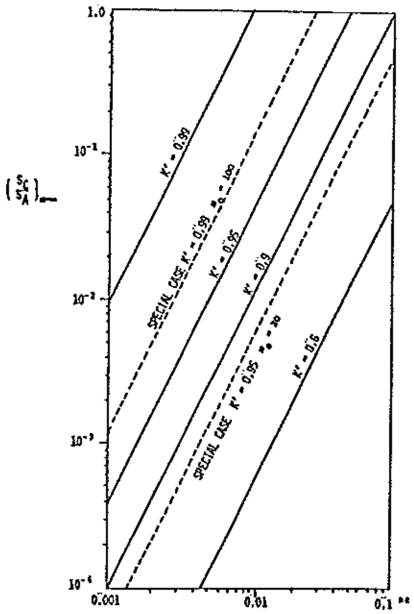

FIG. 3 (b)

4. TECHNIQUE FOR MINIMISING EFFECTS OF SECONDARIES

It is evident that for devices with poor transfer efficiencies or for devices with a large number of stages, the required value of K for a given improvement in signal to noise ratio may not be attained if S_B/S_A is to be kept sufficiently low, say below the built up noise level in that stage. In a practical situation some way has to be found of maintaining a low S_B/S_A ratio whilst keeping K as high as required.

CTD delay lines differ from conventional analogue delay lines in one important respect. They are sampled data devices and hence adjacent signals are contained in adjacent elements of the CTD, precisely defined in time and synchronised to some clock waveform. Similarly the secondary and subsequent signals resulting from one primary are also precisely defined and can therefore be manipulated electronically.

Improvements may be obtained by sampling the input only once every two clock cycles so that an element containing the primary signal is always followed by an initially empty element. As integration proceeds, secondary signals will grow in the empty elements and tertiaries in following elements. As the output of the CTD, the primary is sampled and held, and added to the secondary to produce the output signal. In this way it is the growth of the tertiary that is the limiting factor rather than the secondary. As the tertiary can, for lower values of K and $n\varepsilon$ be an order of magnitude smaller than the secondary this allows a significantly higher value of K to be used and hence a higher signal to noise ratio improvement.

Equations 10, 11, 12 and 13 give

$$\frac{S_C}{S_A} = \frac{n^2\varepsilon^2}{2(1-K')} \cdot \left(\frac{1+K'}{1-K'}\right) \qquad (15)$$

and

$$\frac{S_B}{S_A} = \frac{n\varepsilon}{(1-K')} \qquad (16)$$

There is a decrease of spurious signals by a factor of $\frac{n\varepsilon}{2} \cdot \left(\frac{1+K'}{1-K'}\right)$. Little improvement occurs if the effective loop gain, K', is large enough to be comparable with $(1-n\varepsilon)$ but smaller loop gains give considerable improvement. For example with $n\varepsilon = 10^{-3}$ and $K' = 0.99$ $\frac{S_B}{S_A} = 10^{-1}$ for the simple system whereas a 10-fold improvement occurs in the modified system. No improvement occurs for $K' = 0.999$ but for $K = 0.9$ approximately 100 times improvement would occur.

The dependence of the spurious

signal suppression on loop gain can be removed if the secondary signal in the nominally empty element is returned to zero after each pass through the CCD. The improvement factor over the simple system will be discussed in the oral presentation.

A third but slightly more complicated modification of the simple integrator is illustrated in Fig. 4. The input pulses to the CTD are sampled, inverted, multiplied by a factor $\beta(<1)$, held for one clock period and then added to the input. Assuming that the spurious signal smearing is a linear process the total signal output will be the sum of the two output signal trains shown in Fig. 4. Although the primary is unaffected, the smeared outputs will tend to cancel. Again the improvement factors will be discussed in the oral presentation.

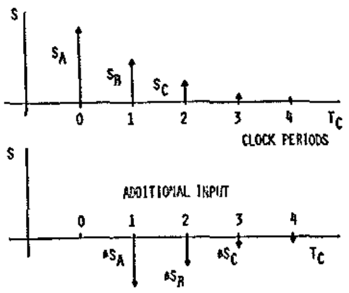

FIG. 4

Outputs after n circulations from the integrator resulting from (a) unit height pulse train (b) the unit height pulse train delayed by one clock period, inverted and multiplied by a factor β.

5. DISCUSSION

A detailed analysis of the build up of spurious signals in CTD integrators has been considered. Computed values of the relative magnitude of primary, secondary and tertiary outputs from the integrator have been presented as functions of K, the loop gain, and $n\varepsilon$, the overall transfer inefficiency of the CTD.

The growth of secondaries is more severe for higher values of K. As higher K gives increased signal to noise ratio improvement, the limitation imposed on K by the necessity of keeping the secondaries below a certain level also reduces the possible improvement in signal to noise ratio.

The limitation on loop gain will be set by the dynamic range required for the input signal. If the smallest detectable signal after integration is equal to the noise voltage at the output, the ratio of primary and spurious signals must be greater than desired dynamic range in order that a large preceding primary signal does not leave a secondary signal in excess of the output noise.

Three techniques were suggested for reducing the effect of secondaries and hence permitting a higher loop gain to be used for a given value of $n\varepsilon$. The first technique, basically the addition of the primary and secondary signals to give the output signal, is simple and effective for lower K and $n\varepsilon$ values. It also has the advantage of improving the signal to noise ratio even if K is not increased.

The second technique reduced the spurious signals by periodically removing the secondaries while the third technique continuously cancelled the secondaries. The improvement in both is equally effective at all values of loop gain and particularly so at low $n\varepsilon$. The second technique is 4 times less effective at reducing smeared signals than the more involved third one. The third technique involves some loss in signal to noise ratio improvement which reduces that gained from the increase in K that it makes possible.

In the analyses discussed, the signal transfer processes within the CTD have been assumed linear with signal amplitude. Normally this is a good approximation, but in this case signal amplitude may vary over several orders of magnitude, from that of the built up primary to the noise level. Consequently it may no longer be justified to use the linear approximation for very high gain systems. In practice the performance of CCD video integrators was found to be limited by non-linearities associated with the input to the device. These prevented very accurate measurements of secondary growth from being made and also restricted the operation of the integrator to low loop gains where the growth is small.

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