Variability limits the advantage of a photo diode's zero bias operation

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Abstract
In the dark a photodiode is a passive structure. At zero bias it thus exhibits zero leakage (dark) current. This feature is known since ever; in hybrid photovoltaic infrared detectors one exploits it to minimize dark current. In monolithic CMOS or CCD sensors this "trick" is often not applicable as the design freedom to bring the bias voltage at zero volts does not exist.

Note that it is not the dark current itself that we want to reduce, but its related noise and spatial non-uniformity. The goal is thus not to reduce the dark current to zero, on average, but to minimize the spread of the effect of dark current. In this paper we will point out that there *is* a lower limit on the improvement due to variability of the dark current and of the pixel's circuit parts.
The essence of this paper: is it worthwhile doing effort to operate photodiodes at or near zero-bias? At first sight, reducing the photodiode's bias voltage to zero is the wonder solution for dark current.

The message is: *Yes*, dark current can be greatly reduced; DSNU and DCSN may decrease too. And *No*, the gain is by far not what you hoped for. In some not-so-extreme operation conditions, it does even more harm than good.

We derive formulas for the basic behavior, which allow estimating the best temperature/integration time/bias voltage working point. These formulas are just tools to understand dependencies. Real prediction requires variability aware analog transient and noise transient simulations.

1 Pixel operation near zero bias

1.1 Leakage current / voltage relation

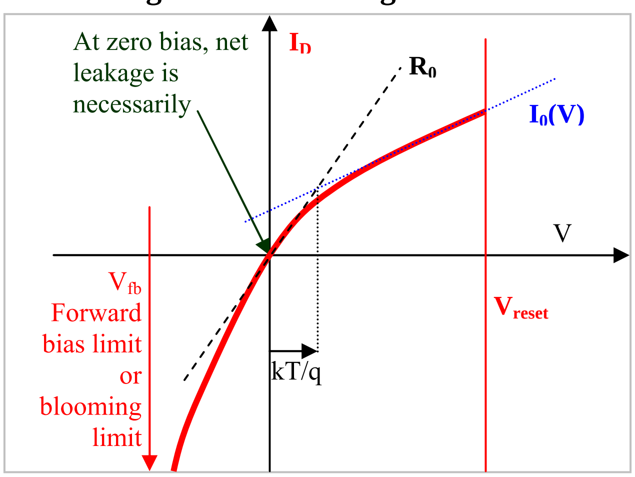

Figure 1 Dark current / voltage relation of a photodiode (arbitrary curve).

The dark current obeys

$$I_D(V) = I_0(V)\cdot\left(1-\exp(\frac{q.V}{k.T})\right) \quad (1)$$

which yield an expression for R_0 as

$$\frac{1}{R_0} = \frac{\partial I_0(V)}{\partial V} + \frac{q\cdot I_0(0)}{kT} \quad (2)$$

$$I = I_{ph} + I_D(V) \approx I_{ph} + \frac{V}{R_0} \quad (3)$$

1.2 Photo response near zero-bias

$$Q_D = (V_{reset} - V) * C = \int I_D(V)\partial t \quad (4)$$

The integral of (1) and (2) expression fit in (4) and yield:

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$$(V_{reset} - V) * C = \int (I_{ph} + I_0(V) \cdot \left(1 - \exp(\frac{q.V}{k.T})\right) dt \approx \int (I_{ph} + \frac{V}{R}) dt \tag{5}$$

This is a differential equation describing V(t). The solution is (simplifying with a constant R or $I_0(V)$):

$$V(t) = (V_{reset} - V_\infty) * \exp(-\frac{t}{RC}) + V_\infty \quad \text{or} \quad V(t) = (V_{reset} + R * I_{ph}) * \exp(-\frac{t}{RC}) - R * I_{ph} \tag{6}$$

For infinite R, this simplifies to the classic photoresponse expression for the integration of photocurrent:

Take the solution $V(t_{\text{int}}) = (V_{reset} + R * I_{ph}) * \exp(-\frac{t_{\text{int}}}{RC}) - R * I_{ph}$ found here above and let's agree that we shall avoid the regime where the behavior becomes non-linear as function of time, or t_{int} << R.C. Then:

$$V(t_{\text{int}}) = (V_{reset} + R * I_{ph}) * (1 - \frac{t_{\text{int}}}{RC} + \left(\frac{t_{\text{int}}}{RC}\right)^2) - R * I_{ph} \tag{7}$$

How to interpret (7)? We learn that the photoresponse remains truly linear (caveat: this linearity falls with the assumption that R and C are linear). Integration starts at V_{reset}, but V_{reset} will seemingly shift towards zero as t_{int} becomes significant compared to RC.

The photoresponse itself will decrease as t_{int} becomes significant compared to RC:

Photoresponse in terms of V/A: $\frac{dV(t_{\text{int}})}{dI_{ph}} = \frac{t_{\text{int}}}{C} * \exp(-\frac{t_{\text{int}}}{RC})$ or simplified: $\frac{dV(t_{\text{int}})}{dI_{ph}} = \frac{t_{\text{int}}}{C} * (1 - \frac{t_{\text{int}}}{RC})$ (8)

2 Spatial non uniformity math

Spatial variability of V_{reset}, R and C will translate into spatial offset and gain non-uniformity. In this paragraph the symbol σ() denotes the *spatial, pixel-to-pixel,* non-uniformity of the expression between brackets.

Caveat: the results are based on the linearized relation of Figure 5. This approximation is only valid when operating very close to zero bias.

2.1 Variability of V_{reset}

Take (6) and make it dark. Pixel to pixel variability of V_{reset} will enter the raw pixel voltage V. On-chip CDS or DS can subtract the reset voltage, but will not cancel all non-uniformity due to V_{reset}.

$$V(t_{\text{int}}) - V(t=0) = V_{reset} \cdot \left(\exp(-\frac{t_{\text{int}}}{RC}) - 1\right) \tag{9}$$

I.e. spatial Variability of V_{reset} enters the pixel signal as an offset non-uniformity (ONU).

$$ONU_{VR} = \sigma(V_{reset}).\left(1 - \exp(\frac{t_{\text{int}}}{RC})\right) \tag{10}$$

2.2 Variability of the photo diode leakage resistance R

Take (7). After CDS or DS and keeping only first order terms in R it becomes:

$$V(t_{\text{int}}) - V(0) = -\frac{1}{R} \cdot \frac{V_{reset} . t_{\text{int}}}{C} + \frac{1}{R} \cdot \frac{I_{ph} . t_{\text{int}}}{C} \cdot \left(\frac{t_{\text{int}}}{C}\right) \tag{11}$$

Differencing to R yields the sensitivity to variability of the signal voltage to variation of R. The first term results in an offset non-uniformity (ONU) that is proportional to t_{int} and thus behaves mathematically as the classic DSNU. The second term is a spatial gain non-uniformity (GNU) that is proportional to I_{ph}, hence it behaves mathematically equal to classic PRNU, *yet* the error is not a pixel constant as with classic PRNU, but a factor that grows linearly with (t_{int}). These offset and gain non-uniformities are mathematically:

$$ONU_R = \frac{\sigma(R)}{R} \cdot \frac{V_{reset} . t_{\text{int}}}{R.C} \quad \text{and} \quad GNU_R = \frac{\sigma(R)}{R} \cdot \frac{t_{\text{int}}}{R.C} \tag{12}$$

The spatial non-uniformity of the leakage current or leakage resistance translates to a form of PRNU. As a rule of thumb this formula teaches that, in order to keep this contribution below the already existing 1..2% PRNU, we need $PRNU_{@LARGE\_BIAS} > \frac{\sigma(R)}{R}.\frac{t_{\text{int}}}{R.C}$. As σ(R)/R is close to unity, this means that one should operate the image sensor in practice in the regime t_{int} smaller than a few % of R.C. This means that image quality may severely degrade at long integration times, exactly the opposite of what one tried to improve with lowering the bias, unless software PRNU correction can be applied.

2.3 Variability of the photo diode capacitance C

Starting from (11) one can now difference to C, yielding:

$$GNU_C = \frac{\sigma(C)}{C} \quad \text{i.e. the classic PRNU, and} \quad ONU_C = \frac{\sigma(C)}{C}.\frac{V_{reset}.t_{\text{int}}}{C.R} \tag{13}$$

3 Dark current shot noise near zero bias

Usually the temporal noise associated to dark current is simplified to be dark current shot noise. DCSN charge is the square root of the number of electrons dark current. Dark current $I_D(V)$ near zero bias, from [3]:

$$I_D(V) = I_0(V) \cdot \left(1 - \exp(\frac{q.V}{k.T})\right) \tag{14}$$

Which consist of the "reverse bias" $I_0(V)$ dark current, itself voltage dependent, and an opposite diffusion current that becomes important near zero bias (V<kT/q). Both terms independently exhibit shot noise. Thus, although the net I_D is zero at zero bias, one has two independent sources of shot noise with spectral density:

$$S_I(f) = 2.q.I_0(V) + 2.q.I_0(V) \cdot \exp(\frac{q.V}{k.T}) \tag{15}$$

Assuming that t_{int} is always far below the bandwidth of the shot noise then

$$Q_{DCSN} = \sqrt{\frac{I_0(V).t_{\text{int}}}{q} \cdot \left[1 + \exp(\frac{q.V}{k.T})\right]} \tag{16}$$

Observations on this formula:

1. The formula predicts that DCSN steeply increases at slight forward bias. This applies equally if this forward bias is due to photocurrent. In that case the photon shot noise power can double as if the photodiode were a photoresistor.
2. In an idealized photodiode $I_0(V)$ is a constant; then DCSN at zero bias is $\sqrt{2}$ times the large bias value. Real diodes might not obey this simplification
3. $I_0(V=0)$ is in fact an unmeasurable value. In the remainder of this text we assume that $I_0(0)$ is known.
4. The formula is in no way exact, as during integration, the diode voltage will travel from V_{reset} towards 0.
5. At large bias (V>>kT/q) the expression becomes equal to the classic expression for DCSN.

4 Summary

Is it worthwhile doing effort to operate photodiodes at or near zero-bias? The real goal of reducing dark current is reducing DSNU and DCSN. If dark current becomes zero on the average, but has large spatial non-uniformities or temporal noise as side effects, then we have gained nothing.

Table 1. In the following table we compare the photodiode related contributors for DSNU, PRNU and DCSN. Note that the low bias operation involves more but likely smaller contributors.

Contributor	High bias operation	Low /zero bias operation

[3] A. van der Ziel, "Noise in Solid State Devices and Circuits", John Wiley & Sons, New York, New York, p. 93-95, 1986

DSNU-like [mV_{RMS}]		$ONU_{VR} = \sigma(V_{reset}).\frac{t_{int}}{RC}$
	$ONU = \sigma(I_D).\frac{t_{int}}{C}$	$ONU_R = \frac{\sigma(R)}{R}.\frac{V_{reset}.t_{int}}{R.C}$
		$ONU_C = \frac{\sigma(C)}{C}.\frac{V_{reset}.t_{int}}{C.R}$
PRNU-like [$\%_{RMS}$] Only the fraction due to the photodiode	$GNU = \frac{\sigma(C)}{C}$	$GNU_C = \frac{\sigma(C)}{C}$
		$GNU_R = \frac{\sigma(R)}{R}.\frac{t_{int}}{R.C}$ (*)
Dark current shot noise DCSN [e^-_{RMS}]	$Q_{DCSN} = \sqrt{I_D.t_{int}/q}$	$Q_{DCSN} = \sqrt{\frac{I_0(V).t_{int}}{q}\cdot\left[1+\exp(\frac{q.V}{k.T})\right]}$

4.1 Example

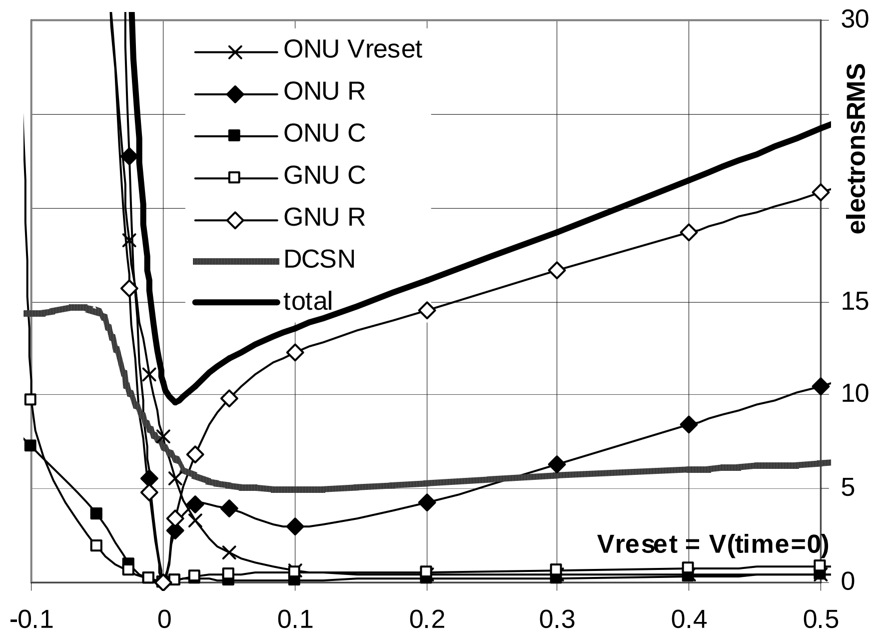

*Figure 2 ONUs, GNUs and DCSN in electrons$_{RMS}$ based on the formulas in table 1, for following parameters and operating conditions: dark, , t_{int}=10sec,σR/R=50%$_{RMS}$, $I_0(V)$=[2e$^-$/s]*1+V/500mV, σC/C=2%$_{RMS}$, σV_{reset}=10mV$_{RMS}$, C=1fF=160 uV/e$^-$,*

The optimal operating point is a slight reverse bias; if all spatial non-uniformity can be calibrated the optimal operating point depends solely on DCSN, V=1V in this example.

5 Conclusions & recommendations

1. Zero bias or near zero bias operation yields a significant reduction of the dark current but just a limited reduction of dark current induced noise and non-uniformity.
2. Its associated spatial non-uniformity effects behave like DNSU and time/temperature dependent PRNU.
3. A serious limitation is that the non-uniformity of the leakage resistance translates to an excess PRNU that becomes dominant when the integration time becomes longer than a few % of the auto-saturation time. However this limitation should not stop one to pursue operation at lower bias.
4. Another important issue is the low full well charge; this may be solved by special technology enhancements or by living with the high photon shot noise. One must anticipate having to work with very low full well charges, in the order of a few 1000 electrons or less
5. When we assume that all effects than can be calibrated are calibrated, in the end we will remain stuck with only the dark current shot noise contribution. In order to avoid a steep increase of shot noise due to forward diffusion, one must stay clear of zero bias operation by a few kT/q.