

# A Study for Image Pickup over Nyquist Rate Using Digital Signal Processing

Takayuki Kimura, Noboru Takatsuka, Takanori Arano, and Hiromitsu Shiraki

Department of Systems Engineering, Faculty of Engineering, Ibaraki University  
Nakanarusawa, Hitachi, Ibaraki 316-8511, Japan

Tel: +81-294-38-5201 Fax: +81-294-38-5272

Email: tkimura@hcs.ipc.ibaraki.ac.jp

## Abstract

The resolution progress of an image sensor by digital signal processing was simulated on computer. By using this method, high-resolution images that have a high-spatial frequency beyond the Nyquist frequency can be obtained. The simulated imaging devices are the area charge-coupled device (CCD) image sensor so that after simulation, the reproduced images have frequency components that are twice as high as the Nyquist frequency. Therefore, it is possible to reproduce high-resolution images that are twice as high as the conventional low-resolution CCD image sensor by using this method.

## 1 Introduction

Charge coupled devices (CCD) have been successful commercially and their cells have reached 2 Mega pixels. However, the number of stored electrons have decreased and the images have degraded since their cell size have become smaller. The resolution and image quality have a trade-off relationship. Kiuchi *et al.*<sup>1,2</sup> reported a method that consisted of optical low-pass and band-pass filters and digital signal processing to solve this dilemma. But their experiment was only adapted to one-dimensional imaging and was not completed. In this work, Kiuchi's method is extended to two-dimensional imaging. First, the principle of two-dimensional imaging was considered. Then, a processing program was programmed on a personal computer. The program consisted of a fast Fourier transform (FFT), an inverse FFT (IFFT), a FFT filter, and a resampling module. Using this program, the resolution enhancement of the CCD image sensor was simulated, and high-resolution images that included a frequency signal higher than the Nyquist frequency was reproduced.

## 2 Principles

### 2.1 The case of one-dimensional signal

Principles of this method are shown in Fig. 1. Spatial frequency components of the input image that are higher than the Nyquist frequency generate replicants. On the conventional CCD image sensor, the spatial frequency components of the input image are limited to being lower than Nyquist frequency by an optical low-pass filter to prevent the generation of replicants. In this method, spatial frequency components of the input image are partitioned by optical filters into a group consisting of smaller bandwidth signals. (Fig. 1 ②). These partitioned images are individually sampled at a spatial sample-rate determined by the photodiode cell-pitch of a CCD image sensor. At this time, the spatial frequency components that are higher than the Nyquist frequency generate replicants at lower spatial-frequency components. Sampled images by CCD image sensor are all transformed to spatial frequency components lower than the Nyquist rate, respectively (Fig. 1 ③). These images are converted to frequency components by fast Fourier transform. Individually handled components

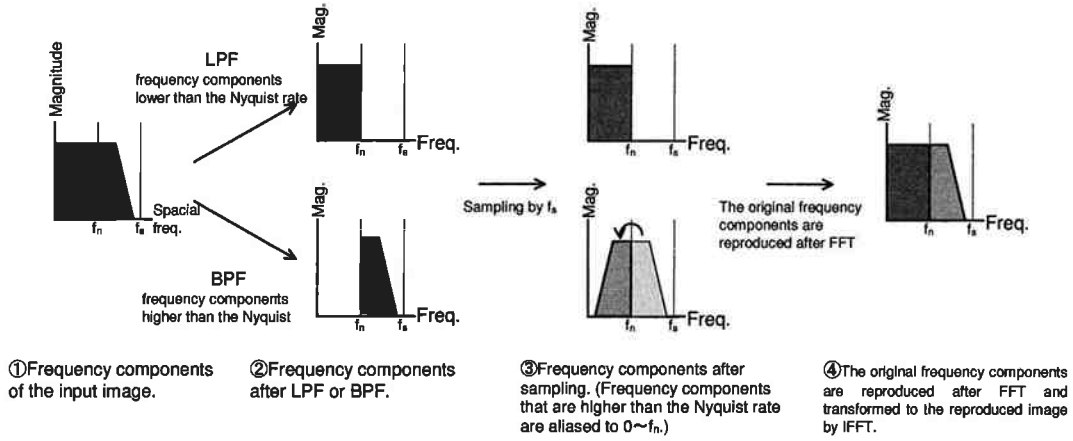


Fig. 1: Principles of the resolution progress of the image sensor by digital signal processing in the case of linear CCD.

are combined, and one original frequency component is reproduced (Fig. 1 ④). Finally, these reproduced components are transformed to the time-domain signals by IFFT.

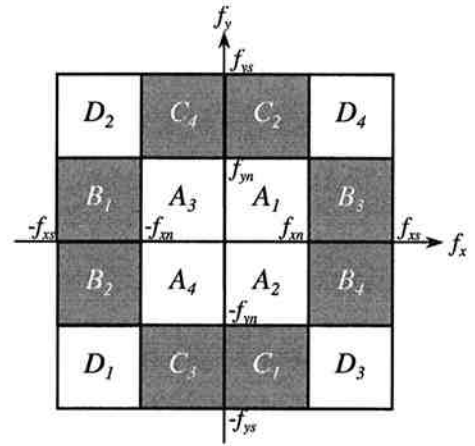
## 2.2 The case of a two-dimensional signal

The principle mentioned above can be extended to two-dimensional images. Equation (1) shows the relation between the original signal and its replicants.

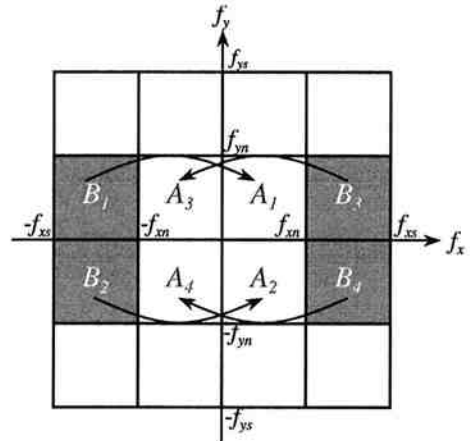
$$C'_{mn} = \sum_{m+tP=-\infty}^{\infty} \sum_{n+uQ=-\infty}^{\infty} C_{m+tP, n+uQ} \quad (1)$$

$t, u$ : natural numbers

$C'_{mn}$  represents the Fourier coefficient.  $P$  and  $Q$  mean the numbers of horizontal and vertical pixels, respectively. This equation also shows the two-dimensional sampling theorem and determines the relation between the original image and the aliased image (Fig. 2). Figure 2(a) shows the frequency components of an original image that are twice the resolution as the Nyquist frequency. In this figure, the original image has frequency components from  $-f_{xs}$  to  $f_{xs}$  in horizontal and  $-f_{ys}$  to  $f_{ys}$  in vertical. This image is sampled by  $f_{xs}$  and  $f_{ys}$ , and area  $B_x$ ,  $C_x$  and  $D_x$  alias to area  $A_x$ . If these replicants ( $B_x$ ,  $C_x$  and  $D_x$ ) are individually aliased to area  $A_x$ , as shown in Fig. 2(b), then signals can be obtained without distortion.



(a) Relation between area  $A$  and the others.



(b) Aliasing example of area  $B$ .

Fig. 2: The relation between the original and replicant image. Area  $B$ ,  $C$ , and  $D$  with the same subscript number is aliased to area  $A$ .

### 3 Simulation Methods

Developments were made and simulations were performed on a personal computer. All simulation programs were programmed by C++. Calculation of two-dimensional FFT of  $512 \times 512$  pixels was executed in less than 5 s. The original input image was the size of  $512 \times 512$  pixels with an 8-bit gray scale. The prepared input image had a circular zone plate (CZP) pattern, and its frequency components were twice as wide as the Nyquist bandwidth of the CCD image sensor. The size of the targeted CCD device was assumed to be  $256 \times 256$  pixels. Optical filters were simulated by the two-dimensional FFT filter. The calculated results are shown as an image without modification. Images in the frequency-domain signals are obtained by calculating the square root of real and imaginary components as the amplitude components, and are shown in logarithmical scale.

### 4 Results and Discussion

The images produced in each process are shown in Fig. 3. In the simulation, the horizontal and vertical spacial sampling rates by CCD image sensor were determined as  $f_{xs}$  and  $f_{ys}$ , respectively, and the Nyquist rates ( $f_{xn}$  and  $f_{yn}$ ) were half of the sampling rate ( $f_{xs}/2$  and  $f_{ys}/2$ ). Figure 3(a) shows the original image which is  $512 \times 512$  pixels and has a CZP pattern. The image was filtered four times with four different passing areas (one low-pass and three band-pass filters), and its bandwidth was the same as the Nyquist bandwidth of the CCD image sensor. Figures 3(b), 3(c), and 3(d) show only the images found in the low-pass filter. These filtered images were individually resampled to half ( $256 \times 256$  pixels) of the original image (Fig. 3(c)). This process corresponds to the sampling of the input image by a CCD image sensor. The resampled images were transformed to frequency-domain signals by FFT (Fig. 3(d)). After FFT, high-spacial frequency components were remapped as replicants to its original frequency components (Fig. 3(e)). And after remapping all the components, these original frequency components were transformed to the original image by IFFT. Figure 4 shows the details of Fig. 3(f). In Fig. 4(a), the image in the lower corresponds to the original image (Fig. 3(a)), and the image in the upper have corresponds to the

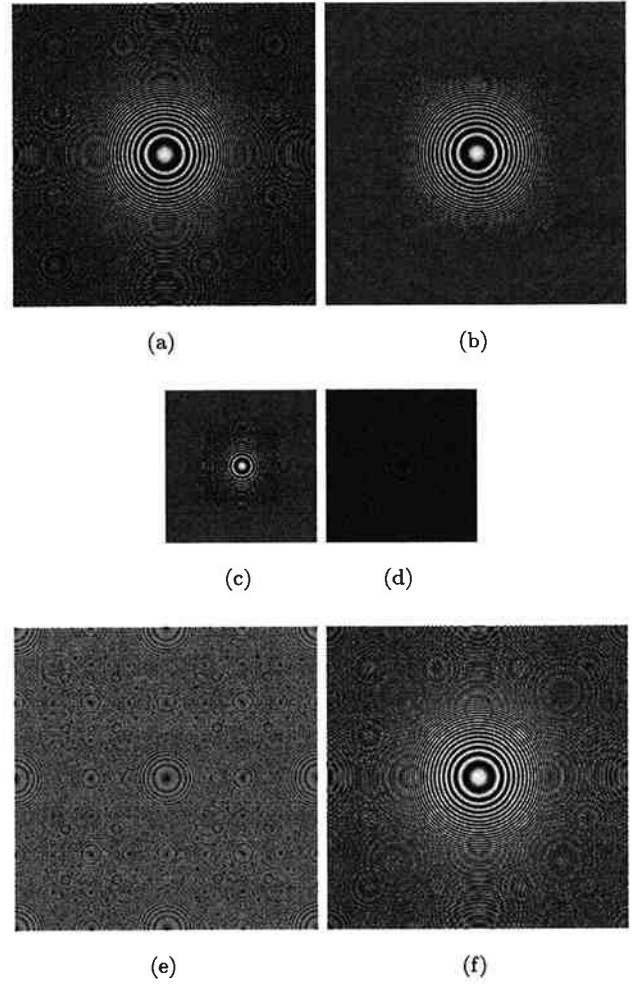


Fig. 3: Images produced in each process. (a) Original CZP. (b) Filtered image by low-pass filter. Higher frequency components than the Nyquist frequency of the CCD image sensor are eliminated. (c) Resampled image of the image (b). (d) Frequency-domain image of the image (c) after FFT. (e) Expanded image from the image (d). (f) Image reproduced by FFT (e).

reproduced image (Fig. 3(f)). From the reproduced figure, frequency components higher than the Nyquist rate of the CCD image sensor are successfully reproduced as shown in Fig. 4(a). Thus, Kiuchi's method is successfully extended to a two-dimensional imaging system and is useful for making a high-resolution imaging system. Though this method is highly advanced, weak low-frequency noises can be recognized as indicated in Fig. 4(b). The frequency of noises is same as the Nyquist frequency of the CCD image sensor. Components of noises run parallel to the x-axis or y-axis. The reason why the noises occurred is due to the mismatch in the image combination process shown in Fig. 3(e).

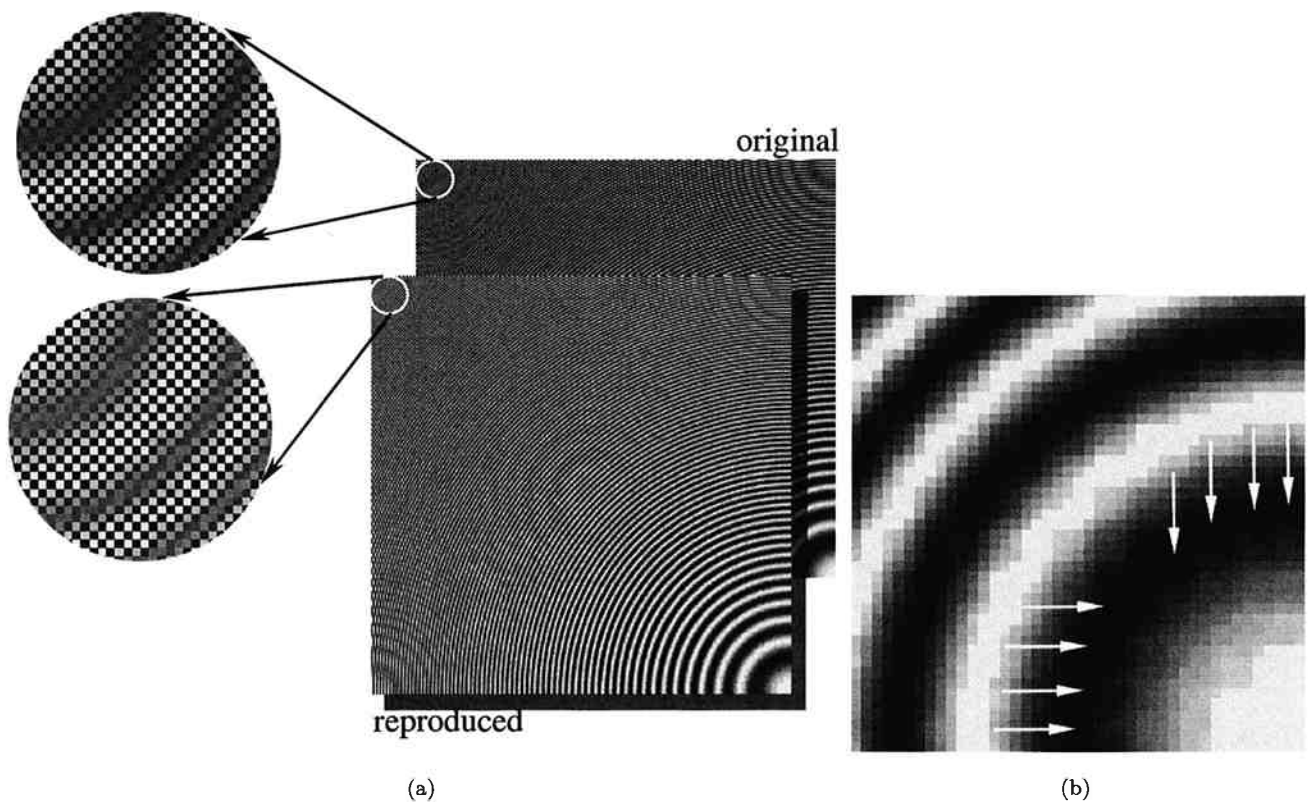


Fig. 4: Details of reproduce images. (a) Comparison between the original image and the reproduced one. Image in the lower half shows the original image and the image in the upper half shows the reproduced one. (b) Magnified reproduced image. White arrows indicate noises produced in the image combination process shown in Fig. 3(e).

Although good results have not yet been obtained, we think that these noises can be reduced by eliminating the mismatched components. Therefore, more filter adjustments and detail considerations about the origin of noises are required.

## 5 Conclusions

The resolution progress of an image sensor by digital signal processing was simulated on a computer. High-resolution images that were twice the resolution as the CCD image sensor were successfully reproduced. By using this method, high-resolution images that have high-spatial frequency beyond the Nyquist frequency could be obtained with the conventional low-resolution CCD image sensor. Therefore, this method is powerful for advancing the image resolution.

## References

- [1] Y. Kiuchi, T. Sakusabe, and S. Saruya; ITE Technical report Vol. 18, No.67, (1994) pp. 1-6. (in Japanese)
- [2] Y. Kiuchi, and T. Sakusabe; 1995 ITE Annual Convention, pp. 59-60 (in Japanese)