DESIGN OF SOLID-STATE IMAGERS

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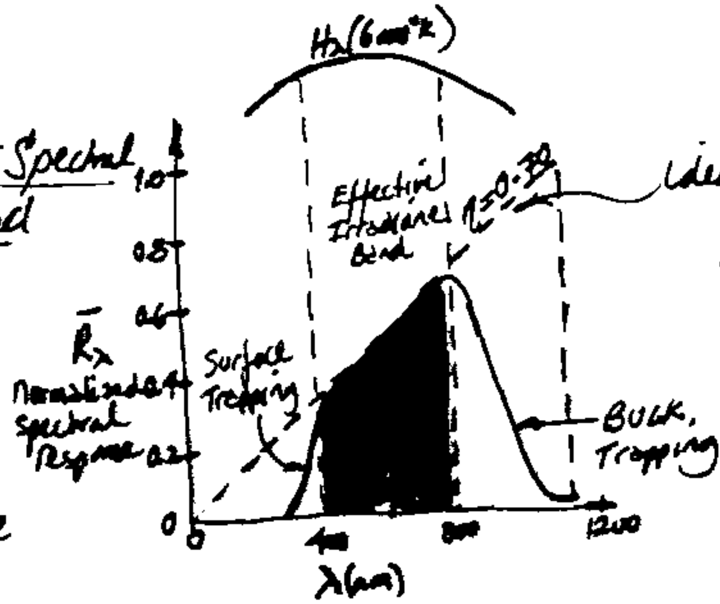
OUTLINE

- Introduction
- Photo Sen[illegible]
- Noise Considerations
- Bandwidth Limitations
- Figure-of-Merit Formulation
- Photo Sensor Array Design
- Conclusions

Photosensor Spectral Response and Responsivity

Silicon photodiode



Ideal Relative Spectral Response

$$R_\lambda = q\frac{\eta(\lambda)}{hc/\lambda} \quad \text{for } \lambda \le \lambda_G$$

$$= 0 \quad \lambda > \lambda_G$$

$$\lambda_G = \frac{hc}{E_G} = 1.1\,\mu m$$

$$\eta \simeq 1 - \left(\frac{n_{Si} - n_{air}}{n_{Si} + n_{air}}\right)^2 = 1 - \left(\frac{3.45 - 1}{3.45 + 1}\right)^2 \simeq 0.70$$

quantum efficiency is almost constant in the 'effective' irradiance band.

Spectral Response

$$R_\lambda = q\frac{\eta(\lambda)}{hc/\lambda} = \frac{J_\lambda}{H_\lambda}\left(\frac{A}{W}\right)$$

current density output per irradiance input

Irradiance

$$H_\lambda = \frac{2\pi c^2 h}{\lambda^5\left(e^{hc/\lambda k T_S} - 1\right)} \left(\frac{W}{m^2 \cdot \mu m}\right)$$

Ideal Planckian Blackbody Source

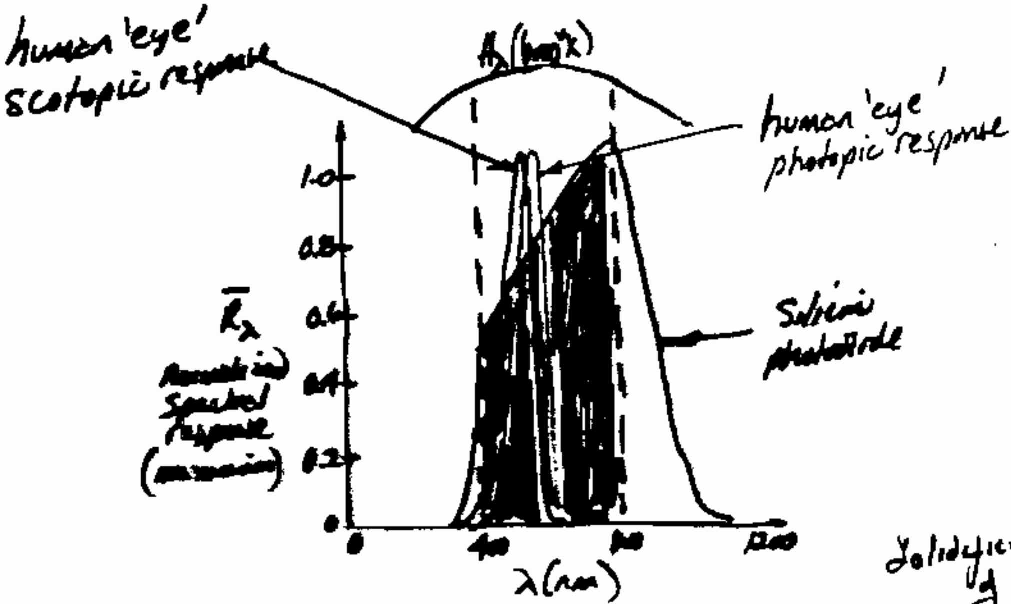
Responsivity

$$R = \frac{J}{H_{eff}} = \frac{\int_{\lambda_1}^{\lambda_2} d\lambda\, R_\lambda H_\lambda}{\int_{\lambda_1}^{\lambda_2} d\lambda\, H_\lambda}$$

Let $x = hc/\lambda k T_S$, $\lambda_2 = 800\,nm$ $\lambda_1 = 400\,nm$ $T_S = 6000°K$ $\eta = 0.7$

$$R = \frac{q\eta}{kT_S} \frac{\int_{x_1}^{x_2} dx\, x^2 e^{-x}}{\int_{x_1}^{x_2} dx\, x^3 e^{-x}} = 0.33\frac{A}{W}$$

Radiometric and Photometric Considerations



Solidification Temperature of platinum

Definition of a Lumen: $60\pi \times 10^4 \frac{\text{lumens}}{m^2} = R_\lambda(\max) \int_0^\infty \bar{R}_\lambda(eye) H_\lambda(2042°K)\, d\lambda$

$R_\lambda(\max)$ = 680 lumens/watt at the spectral peak ($\lambda_m = 556\,nm$) for the photopic Eye Response

note: 1 meter-candle = 1 $\frac{\text{lumen}}{m^2}$ = 1 lux
= 10 $\frac{\text{lumens}}{ft^2}$ = 10 ft-candles

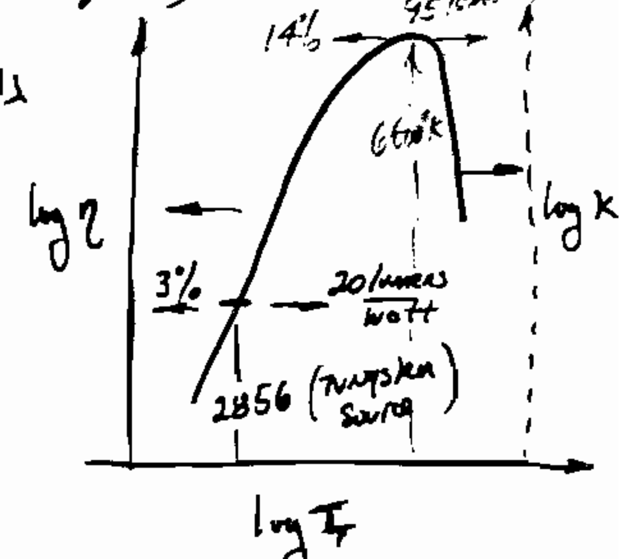
$$F \text{ (luminance)} = 680 \int_0^\infty \bar{R}_\lambda(eye) H_\lambda(T_T)\, d\lambda$$

$$K \text{ (luminosity)} = \frac{F}{\int_0^\infty f(\lambda) H_\lambda(T_T)\, d\lambda} = \frac{680 \int_0^\infty \bar{R}_\lambda(eye) H_\lambda(T_T)\, d\lambda}{\int_0^\infty f(\lambda) H_\lambda(T_T)\, d\lambda} \left(\frac{\text{lumens}}{\text{watt}}\right)$$

cut-off of optical transmission by filters

The luminosity is the ability of the test source to convert watts (radiant power) into lumens (luminescent power).

$$\eta \text{ (efficiency)} = 100 \frac{\int_0^\infty \bar{R}_\lambda(eye) H_\lambda(T_T)\, d\lambda}{\int_0^\infty f(\lambda) H_\lambda(T_T)\, d\lambda}$$



$T_T = 6000°K$
$\lambda_2 = 800\,nm$
$\lambda_1 = 400\,nm$
$> K_S = 200 \frac{\text{lumens}}{\text{watt}}$

Photo sensor Sensitivity

$$S = \frac{R}{K_S} = \frac{0.33\ A/W}{200\ (lm/W)} = 1.65 \frac{mA}{lm}$$

Detector Responsivity

$$R_D = R A_D$$

example: $A_D = 15\mu m \times 20\mu m$

$$R_D = 0.33\frac{A}{W}\left(15 \times 20\mu m^2\right) = 0.0984\frac{pA}{nW/m^2}\left(\frac{616\, e^-}{\mu J/m^2}\right)$$

Signal charge

$$Q_S = \underbrace{R_D H_{eff}}_{I_{ph}}\overset{\text{exposure time}}{\tau} = R_D E_{eff}$$

exposure density ($\mu J/m^2$)

detector responsivity ($pC/\mu J/m^2$)

photon generated current

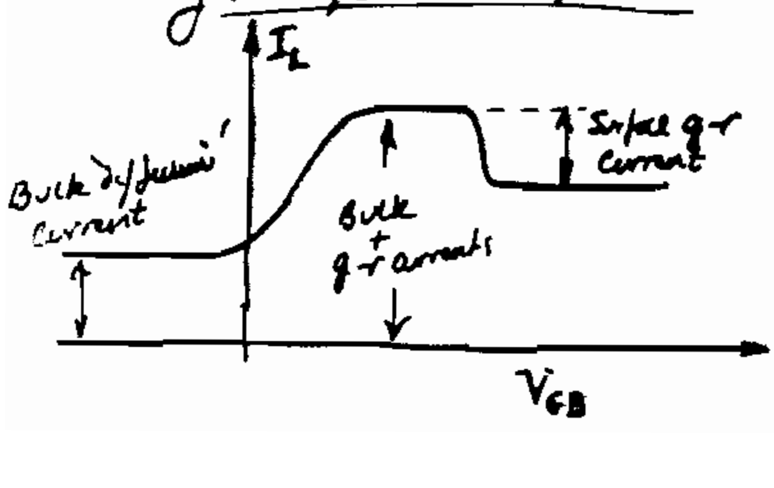
In general, we have the leakage current which adds to the signal.

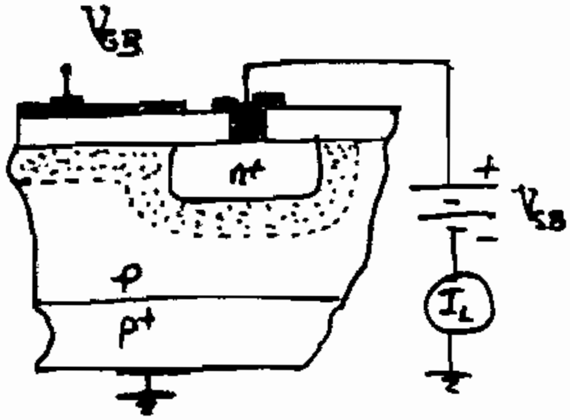
$$Q_S = (I_{ph} + I_L)\tau$$

where,

$$I_L = \underbrace{\frac{q n_i W_j A_D}{\tau_{g-r}}}_{\text{g-r current in bulk depletion region}} + \underbrace{q n_i S_0 A_S}_{\text{g-r current at the surface}} + \underbrace{q\frac{n_i^2}{N_B}\sqrt{\frac{D_n}{\tau_n}}A_D}_{\text{bulk 'diffusion' current}}$$

Test Structure for Determination of Leakage Current Components





Gated Diode Structure

NOISE SOURCES

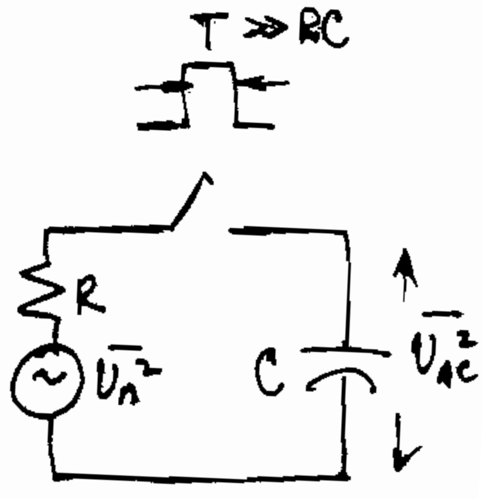
- **Internal**

  (a) **Random Noise** (Time-Variant)
  - **Thermal** or 'Nyquist' Noise
  - **Shot** or 'Schottky' Noise
  - **Generation-Recombination** or '1/f' Noise

  (b) **Fixed Pattern Noise** (**Time**-Invariant)
  - Temperature
  - Responsivity
  - Spectral Response

- **External**
  - Signal Shot Noise
  - Quantization (A/D Converter) Noise
  - Preamplifier ([illegible]) Noise
  - Power Supply Noise
  - Radiation 'Interference' Noise

Nyquist 'Thermal' Noise



Johnson Nyquist Noise

$$\overline{V_n^2} = 4kTR\Delta f$$

$$\overline{V_{nc}^2} = \int_0^\infty \frac{4kTR\,d\omega}{2\pi\left[1+\omega^2R^2C^2\right]} = \frac{kT}{C}$$

or $\boxed{\overline{Q_{nc}^2}\ (\text{Thermal}) = \overline{V_{nc}^2}\,C^2 = kTC}$

in Noise Electrons

$$n = \left(\overline{N_{nc}^2}\right)^{1/2} = \frac{\sqrt{kTC}}{q} = \underline{12.5\sqrt{C(fF)}\ e^-}$$

For example: $C = 25\,fF$

$$n = \left(\overline{N_{nc}^2}\right)^{1/2} \doteq 63$$ e^- r.m.s. noise electrons

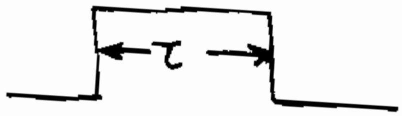
An alternative derivation:

$$\frac{1}{2}kT = \frac{1}{2}C\,\overline{V_{nc}^2}$$ (Equipartition Theorem)

$$\therefore \overline{V_{nc}^2} = kT/C$$

'Shot' Noise

Schottky Formula



$$\overline{I_n^2} = 2 q I_0 \Delta f$$

If we allow electrons to accumulate over an integration time τ, then the r.m.s. charge accumulation becomes,

$$\overline{Q_n^2} = \int_0^\infty \left(\frac{\overline{I_n^2}}{\Delta f}\right) \underbrace{\tau^2 \left(\frac{\sin \pi f \tau}{\pi f \tau}\right)^2}_{|F(\omega)|^2 \text{ of filter function}} df$$

$$\boxed{\overline{Q_n^2} = q I_0 \tau}$$

in Noise Electrons

$$n = \left(\overline{N_n^2}\right)^{1/2} = \sqrt{\frac{I_0 \tau}{q}} \doteq 2.5 \sqrt{I_L(pA)\, \tau(ms)}\ e^-$$

example: $I_L = 2\,pA \quad \tau = 10\,ms$

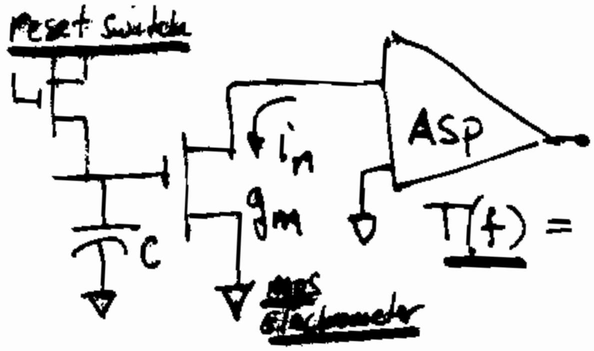
$n = 11\ e^-$ r.m.s noise electrons

Photon Shot Noise

$$\boxed{\overline{Q_n^2} = q R_D E_{eff}}$$

Gate Referred Noise in a MOS Electrometer Amplifier

A MOS transistor is often employed as the collection node for signal and noise charge in a sensor IC. The gate referred noise is calculated as follows:



$\overline{T(f)}$ = normalized transfer function

The noise current i_n is referred to the input to create a noise charge

$$\overline{q_{nG}^2} = \left(\frac{C}{g_m}\right)^2 \overline{i_n^2}$$

If we use the transfer function $T(f)$ of the Analog Signal Processor (ASP), then the noise charge becomes,

$$\overline{q_{nG}^2} = \left(\frac{C}{g_m}\right)^2 \int_0^\infty \left(\frac{\overline{i_n^2}}{\Delta f}\right) [T(f)]^2 df$$

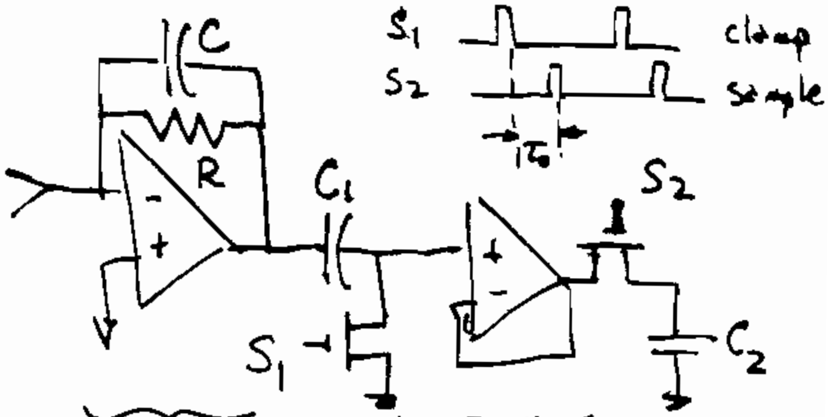
$$= \underline{\left(\frac{C}{g_m}\right)^2 B_{eff} \left(\frac{\overline{i_{n0}^2}}{\Delta f}\right)}$$

where

$$B_{eff} = \text{effective bandwidth} = \underline{\int_0^\infty \left(\frac{i_n^2/\Delta f}{i_{n0}^2/\Delta f}\right) [T(f)]^2 df}$$

For example,

We assume the ASP with the normalized transfer function

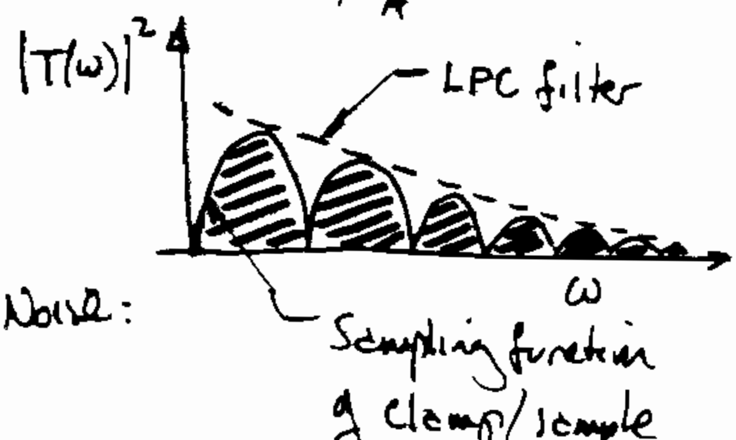


$$T_1 = \frac{1}{1+s/\omega_A} \qquad T_2 = 1-e^{-s\tau_0}$$

$$T = T_1\,T_2$$

$$= \frac{1-e^{-s\tau_0}}{1+s/\omega_A}$$

$$|T(\omega)|^2 = T(j\omega)\,T(-j\omega) = \frac{4\sin^2(\omega\tau_0/2)}{1+\omega^2/\omega_A^2}$$



Case (a)

We assume a 'white' Noise spectrum of Shot Noise:

$$\therefore B_{eff} = \int_0^\infty [T(\omega)]^2\,df$$

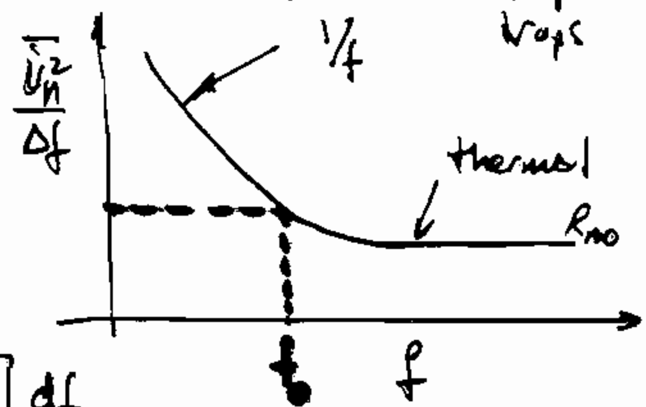
$$= \int_0^\infty \frac{4\sin^2(\pi f\tau_0)}{1+(f/f_A)^2}\,df = \pi f_A\left(1-e^{-2\pi f_A\tau_0}\right)$$

and $$\overline{q_n^2} = 2\pi q I_0 f_A\left(1-e^{-2\pi f_A\tau_0}\right)\left(\frac{C}{g_m}\right)^2$$

$$= q I_0 \omega_A\left(1-e^{-\omega_A\tau_0}\right)\left(\frac{C}{g_m}\right)^2$$

Case (b) We consider a '1/f' noise spectrum caused by interface traps

$f_0 \sim D_{it}$ interface traps



Effective noise resistance

$$R_n = R_{no}\left(1 + \frac{f_0}{f}\right)$$

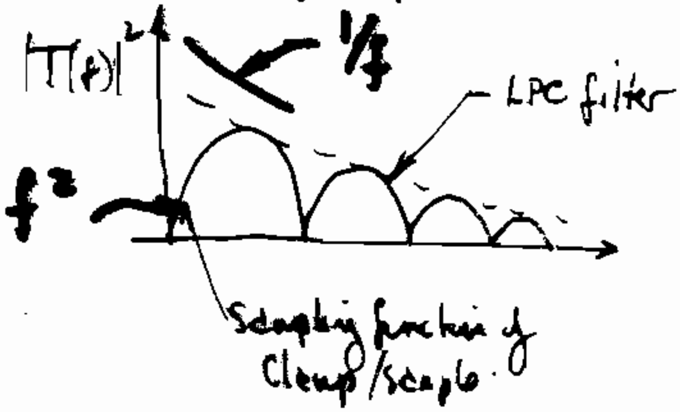
and bandwidth,

$$B_{eff} = \int_0^\infty \left(1 + \frac{f_0}{f}\right)\left[\frac{4\sin^2 \pi f \tau_0}{1 + f^2/f_A^2}\right] df$$

$$= \pi f_A\left(1 - e^{-2\pi f_A \tau_0}\right) + (\pi f_A \tau_0)^2 f_0 \underbrace{\int_0^\infty \frac{\sin^2 x \, dx}{x\left[x^2 + (\pi f_A \tau_0)^2\right]}}_{}$$

1/f noise is suppressed by Clamp/Sample operation

'double zero' of Clamp Sample suppresses noise due to '$\frac{1}{f}$' spectrum



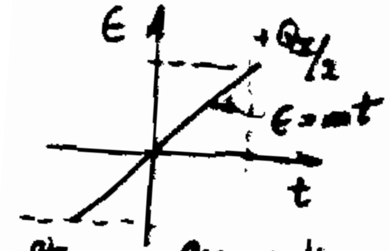
Total Noise Referred to Gate of MOS Transistor

$$\overline{Q_{nT}^2} = \overline{Q_n^2}(\text{Nyquist}) + \overline{Q_n^2}(\text{shot}) + \overline{Q_n^2}\left(\begin{array}{l}\text{gen/rec +}\\ \text{amplifier noise}\end{array}\right)$$

$$= \underbrace{kTC}_{\text{Nyquist}} + \underbrace{q I_0 \tau}_{\text{Leakage 'shot'}} + \underbrace{q R_D E_{eff}}_{\text{photon 'shot'}} + \underbrace{\left(\frac{C}{g_m}\right)^2 B_{eff}\left(\frac{\overline{i_{no}^2}}{\Delta f}\right)}_{\text{gate referred amplifier noise + g-r noise}}$$

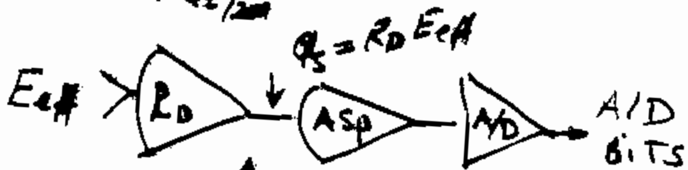
Total Noise Equivalent Signal (NES_{TOT})

The Quantization by the A/D Converter in an Image Sensing System can influence the determination of the noise equivalent signal.

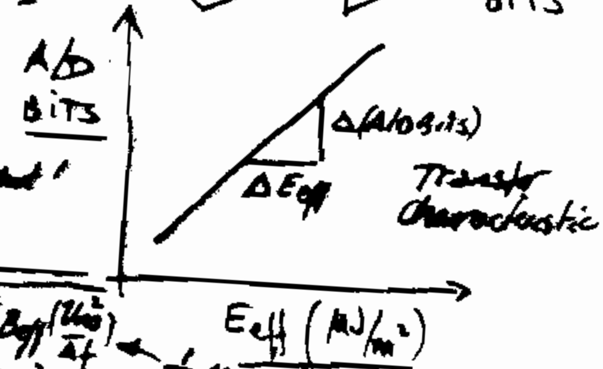


Assume the error varies linearly with time.

$$\overline{E^2} = \frac{\int_{-Q_I/2m}^{Q_I/2m} E^2\,dt}{\int_{-Q_I/2m}^{Q_I/2m} dt} = \frac{m^2 t^3/3 \Big|_{-Q_I/2m}^{Q_I/2m}}{t \Big|_{-Q_I/2m}^{Q_I/2m}} = \frac{Q_I^2}{12}$$



$$\text{Quantizing Interval} \equiv \frac{\Delta E_{eff}}{\Delta(A/D\ \text{bits})} = Q_I$$



$$\frac{S}{N} = \frac{Q_S}{\left[\overline{Q_n^2}(\text{tot})\right]^{1/2}} = \frac{R_D E_{eff}}{\sqrt{\left(\frac{C}{g_m} i_n\right)^2 + kTC + qI_L\tau + C^2 B_{eff}\left(\frac{\overline{U_{no}^2}}{\Delta f}\right) + qE_{eff}R_D + \frac{(Q_I R_D)^2}{12}}}$$

Analog 'Shot'

System

Amp input

$\frac{1}{f}$ MOS electrometer

Photon 'Shot'

A/D Quantization

The Noise equivalent Signal is measured as (Bits)rms for a specified irradiance level (exposure density) and is given as

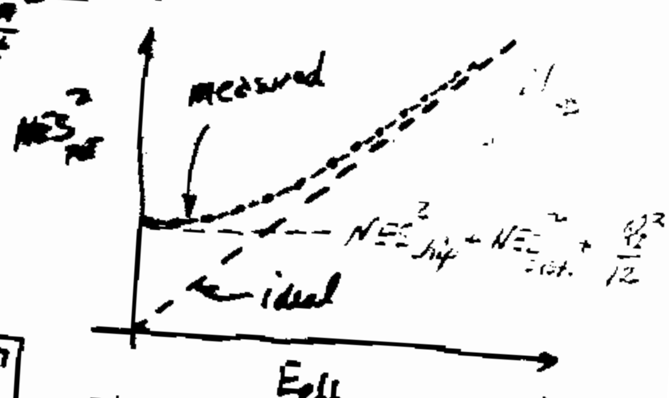
bits variance

$$\boxed{NES_{tot}(\text{meas.}) = Q_I\, B_{rms}} \quad (\mu J/m^2)$$

$\left(\frac{\mu J/m^2}{bit}\right)$

We can define this as the input exposure density at low light levels when the $\frac{S}{N} \ll 1$. Thus, neglecting System and A/D quantization noise, we have

$$\boxed{NES_{Amp} = \frac{1}{R_D}\sqrt{kTC + qI_L\tau + C^2 B_{eff}\left(\frac{\overline{U_{no}^2}}{\Delta f}\right)}}$$



Slope at high exposure densities gives the Detector Responsivity, R_D.

Optimization of the Electrometer Amplifier

$$\frac{S}{N} = \frac{R_0 E_{eff.}}{\sqrt{kTC + qI_0\tau + \left(\frac{C}{g_m}\right)^2 B_{eff}\left(\frac{\overline{i_{no}^2}}{\Delta f}\right)}}$$

where we consider the optimization with respect to NES_0.

mos amplifier

$$C = C_A + C_p \leftarrow \text{parasitic (interconnect + junction)}$$

$$\frac{\overline{i_{no}^2}}{g_m^2 \Delta f} = \frac{\overline{v_{ni}^2}}{\Delta f} = 4kTR_n \quad \left(R_n \simeq \frac{\alpha}{4C_A}\right)$$

From the theory and experiments on '$\frac{1}{f}$' noise.

thus,

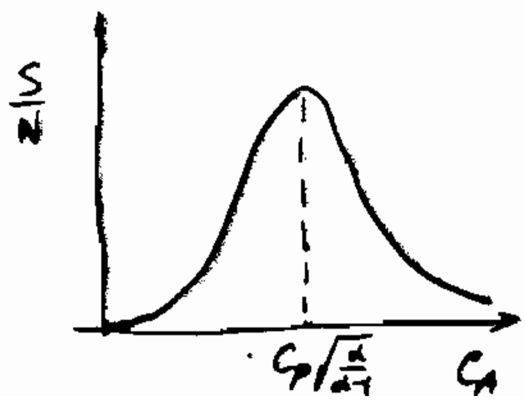
$$\frac{S}{N} = \frac{R_0 E_{eff.}}{\sqrt{kT(C_A+C_p) + qI_0\tau + \frac{(C_A+C_p)^2}{C_A}kT\alpha}}$$

We now maximize the $\left(\frac{S}{N}\right)$ ratio with respect to C_A,

$$\frac{\partial}{\partial C_A}\left(\frac{S}{N}\right) = 0 \quad \text{gives} \quad kT - kT\alpha\left[\frac{C_A^2 - C_p^2}{C_A^2}\right] = 0$$

$$\Rightarrow C_A = C_p\sqrt{\frac{\alpha}{\alpha-1}} \rightarrow C_p$$

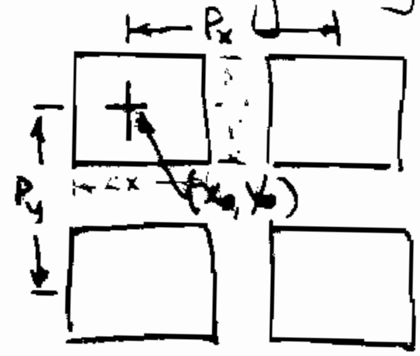
for low Nyquist Noise contributions



Thus, for a given parasitic capacitance, C_p, we should try to design the amplifier, such that $C_A = C_p$.

Bandwidth Considerations

We consider the design of a solid-state imaging array which employs discrete sensors.



We use the concept of a modulation transfer function (M.T.F.) which is the pixel response to a sinusoidal spatial frequency input.

$$H(x) = \begin{array}{c}\text{irradiance of the}\\ \text{input signal}\end{array} = H_0\left(1 + A\cos 2\pi f_s x\right)$$

where f_s is the spatial frequency of the input signal (lp/mm). The maximum spatial frequency which may be processed by the sensor array is $f_s(\max) = \frac{1}{2p}$, which is set by the Nyquist Sampling theorem. The uncertainty in the measurement may be written as,

$$\Delta H(x_0) = H(x_0) - H_m(x_0) = H(x_0) - \frac{1}{\Delta x}\int_{x_0 - \frac{\Delta x}{2}}^{x_0 + \frac{\Delta x}{2}} dx\, H(x)$$

where

$$H_m(x_0) = H_0\left(1 + M\cos 2\pi f_s x_0\right)$$

$$M \triangleq A\,\frac{\sin 2\pi f_s \Delta x}{\pi f_s \Delta x}$$

with a definition of the Modulation Transfer Function (M.T.F.)

$$\boxed{\text{M.T.F.} \triangleq \frac{M(f_s)}{M(0)} = \frac{\sin \pi f_s \Delta x}{\pi f_s \Delta x} = \frac{\sin\left(\frac{\pi \Delta x}{2p}\frac{f_s}{f_s(\max)}\right)}{\frac{\pi\Delta x}{2p}\frac{f_s}{f_s(\max)}}}$$ Geometric M.T.F.

We treat the design of a line array with

$x \longrightarrow$ electrical scan

$y \downarrow$ mechanical scan

$$M.T.F._x = \frac{\sin \pi f_s \Delta x}{\pi f_s \Delta x}$$

$$M.T.F._y = \frac{\sin \pi f_s \Delta y}{\pi f_s \Delta y} \cdot \underbrace{\frac{\sin \pi f_s \overset{\text{velocity}}{\nu} \overset{\text{exposure time}}{\tau}}{\pi f_s \nu \tau}}_{\text{Smear function}}$$

Figure-of-Merit (F.M.)

We formulate a figure-of-merit for a solid state imager as follows:

$$\boxed{F.M. = \frac{S}{N} \times M.T.F.}$$

where we will consider the intrinsic F.M. associated with the chip and the geometrical M.T.F.

Case (1) Noise is determined by the preamplifier and is independent of the sensor area.

$$\therefore \frac{S}{N} \sim \Delta x \Delta y$$

The effective resolution (M.T.F.) is taken as the geometric mean of its components and we write

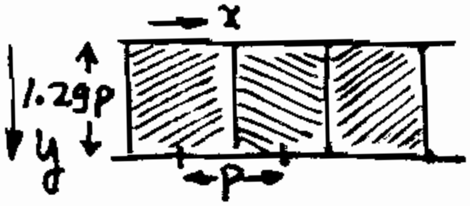
$$\boxed{F.M. = \Delta x \Delta y \left[M.T.F._x \, M.T.F._y \right]^{\frac{1}{2}}}$$

We maximize the F.M. at the Nyquist Limit $f_s = f_{s(max)} = \frac{1}{2p}$ and let $\nu\tau = p$ to conserve bandwidth,

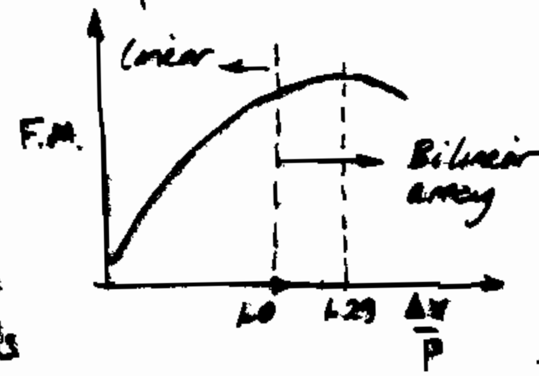
$$\boxed{F.M. \Big|_{f_{s(max)}} = \Delta x \Delta y \left[\frac{\sin\left(\frac{\pi \Delta x}{2p}\right)}{\pi \frac{\Delta x}{2p}} \right]^{\frac{1}{2}} \left[\frac{2}{\pi} \frac{\sin\left(\frac{\Delta y}{2p}\right)}{\frac{\pi \Delta y}{2p}} \right]^{\frac{1}{2}}}$$

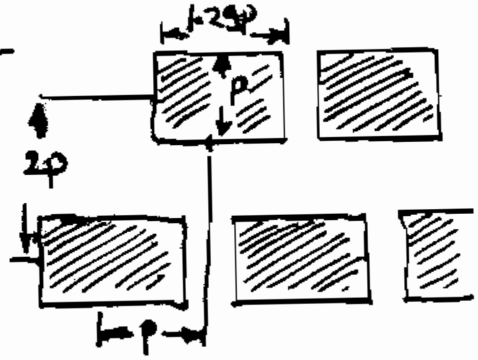
Differentiation with respect to Δy yields,

$$\tan\frac{\pi\Delta y}{2p} = -\frac{\pi \Delta y}{2p} \quad \Rightarrow \quad \Delta y = 1.29p$$



a true linear array with contiguous pixels and a 0P off-set.





a Bilinear Array for maximum Performance with a 2P off-set.

Cartography

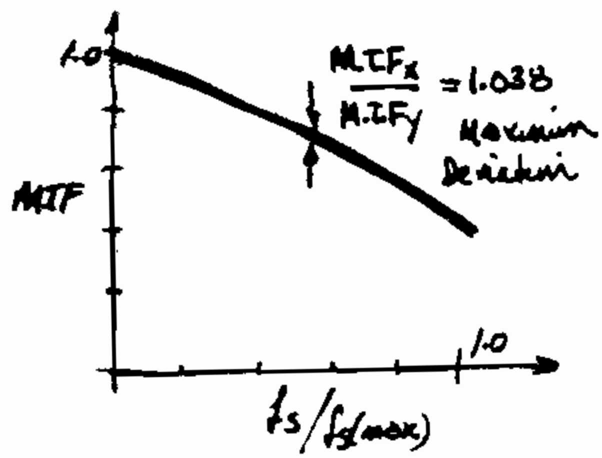
In applications, such as Cartography, equal resolution is required in both x and y directions. We set $MTF_x = MTF_y$ at $f_s = f_s(max)$ and let $\Delta\tau = p$ to conserve bandwidth.

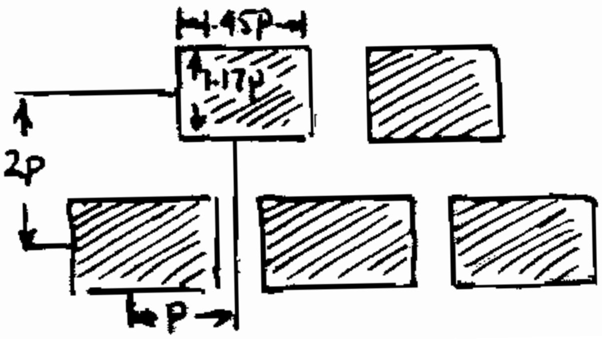
$$\underbrace{\frac{\sin\left(\frac{\pi\Delta x}{2p}\right)}{\frac{\pi\Delta x}{p}} = \frac{2}{\pi}\,\frac{\sin\left(\frac{\pi\Delta y}{2p}\right)}{\frac{\pi\Delta y}{2p}}}_{\text{Constraint}}$$

$$F.M. = \frac{4\Delta x p}{\pi^2}\sin\left(\frac{\pi\Delta y}{2p}\right)$$

We maximize the F.M. subject to the constraint and find

$$\boxed{\begin{array}{ll} \Delta x = 1.45p & \text{preamplifier} \\ \Delta y = 1.17p & \text{Noise Limited} \end{array}}$$





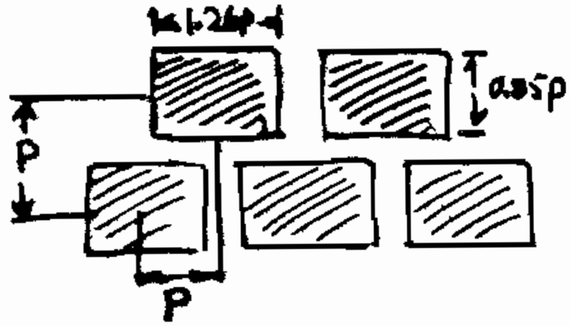
Case(2) If the preamplifier noise is very low, then the noise can be limited by the sensor.

$$\therefore \frac{S}{N} \sim (\Delta x \Delta y)^{1/2}$$

and we find the equal resolution array design becomes.

$$\Delta x = 1.26p \qquad \Delta y = 0.85p$$

Detector Noise Limited

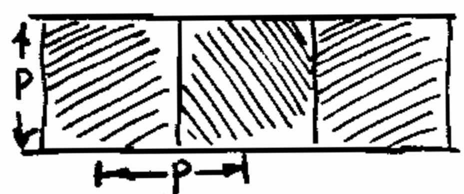


Bilinear Array 1P off-set (Detector Noise Limited)

For the linear array we cannot have equal resolution but we can conserve bandwidth to find,

$$\Delta x = \Delta y = p$$

Detector Noise Limited



Linear Array

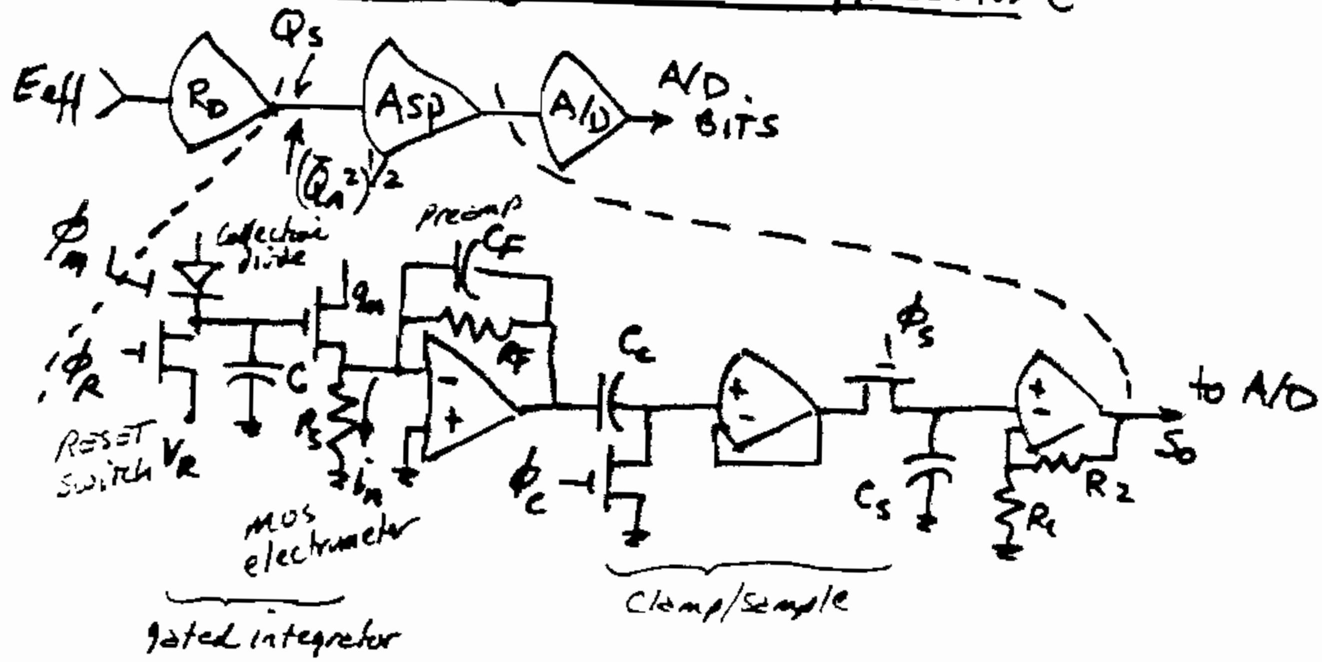
Summary of Design for Line Arrays

Type	Preamplifier Limited	Detector Limited
Maximum Performance Linear Array	$\Delta x = p$ $\Delta y = 1.29p$ 0p off-set	$\Delta x = \Delta y = p$ 0P off-set
Equiresolution Bilinear Array	$\Delta x = 1.45p$ $\Delta y = 1.17p$ 2p off-set	$\Delta x = 1.26p$ $\Delta y = 0.85p$ 1p off-set

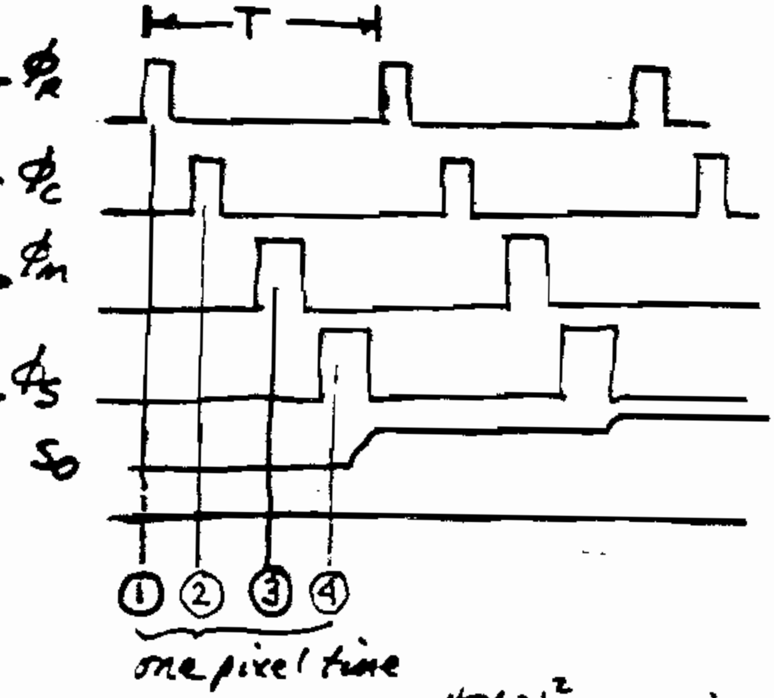
NOISE SUPPRESSION TECHNIQUES

- Integration (storage)
- Differential amplifier (common mode rejection)
- Correlation (e.g. matched filter)
- Synchronous detection (AM demodulation, PLL)
- Adaptive filtering (noise canceller)
- Correlated double sampling (CDS)
- Ring Junction Gate Detection (RJG)

Correlated Double Sampling (CDS) Method for Noise Suppression



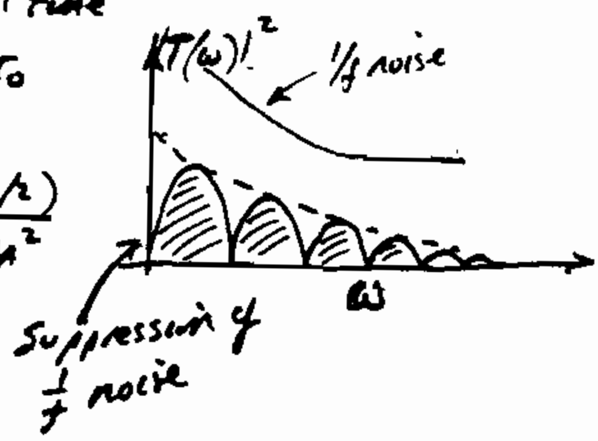
1. Reset C with → ϕ_R
2. Clamp Reset and Noise with → ϕ_c
3. Mux Signal onto Capacitor C with → ϕ_m
4. Sample Signal (less reset noise) with → ϕ_s



$$T_{preamp} = \frac{1}{1 + s/\omega_A} \qquad T_{c/s} = 1 - e^{-sT_0}$$

$$|T(\omega)|^2 = T_{preamp}\, T_{c/s} = \frac{4\sin^2(\omega T_0/2)}{1 + \omega^2/\omega_A^2}$$

- removes kTC Noise
- Suppresses 1/f noise



CONCLUSIONS

- Design of solid-state imagers may be achieved by maximizing the figure-of-merit = (S/N)*(MTF) with respect to the pixel dimensions

- Reset and 1/f noise cancellation may be accomplished with a correlated double sampling technique

- The (S/N) may be maximized by designing the output amplifier capacitance, Ca, to match the parasitic capacitance, Cp